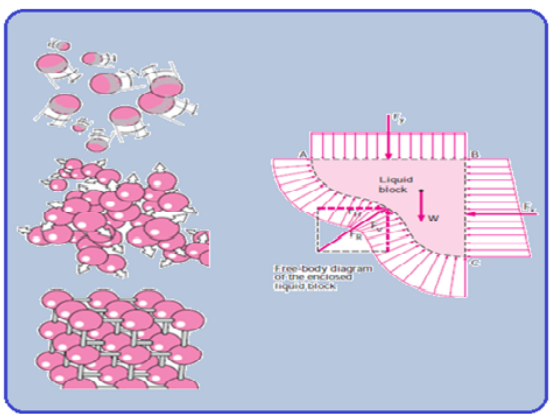
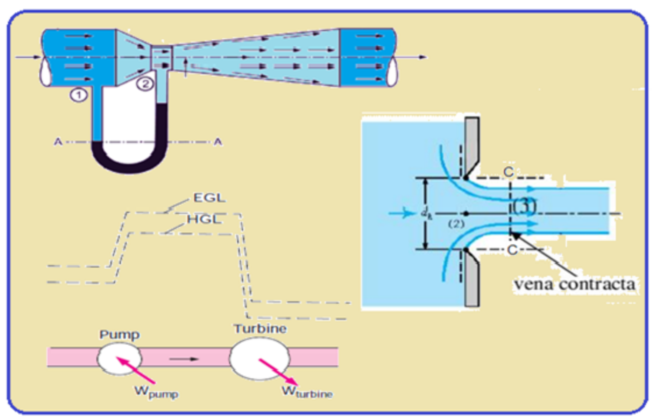
FLUID MECHANICS

Introduction
Statics of Fluids
Kinematics of Fluid Flow





Dynamics of Fluid Flow
Momentum Analysis of Flow Systems
Flow in Pipes
Dimensional Analysis and Similarity

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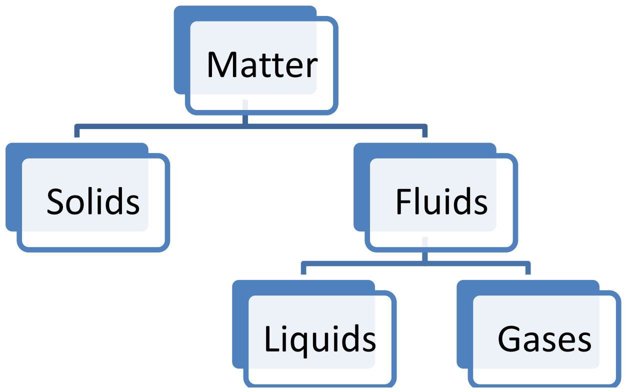
Table of Contents

Chapter 1

INTRODUCTION

1. The Fluids.	*2. Dimensions.*	*3. Units.*	*4. Fluid Properties.*

1-1 The Fluids



As illustrated in Figure 1, the arrangement of atoms in different phases: (*a*) molecules are at relatively fixed positions in a solid, (*b*) groups of molecules move about each other in the liquid phase, and (*c*) molecules move about at random in the gas phase.

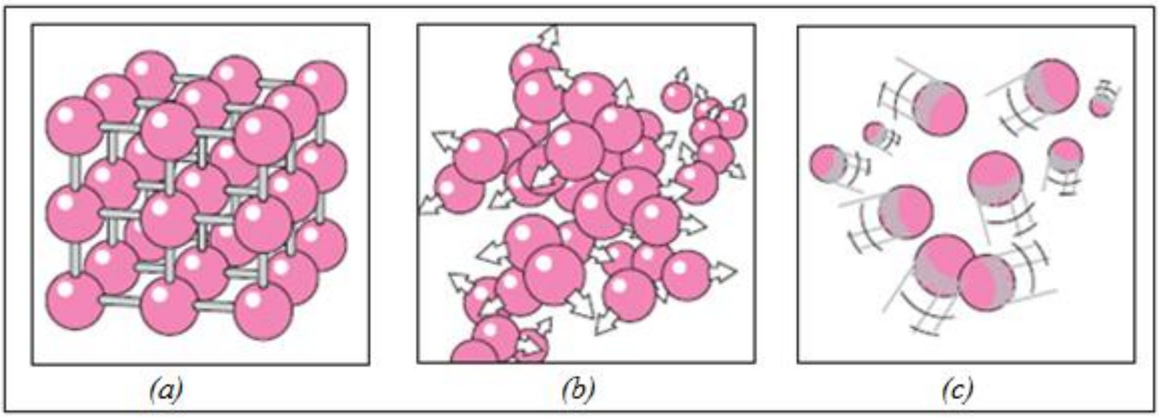


Figure 1. Solids, Liquids, and Gases

The fluid is the substance that deforms continuously when subjected to shear stress, as shown in Figure 2.

Figure 3 shows the normal stress and shear stress at the surface of a fluid element. For fluids at rest, the shear stress is zero and pressure is the only normal stress.

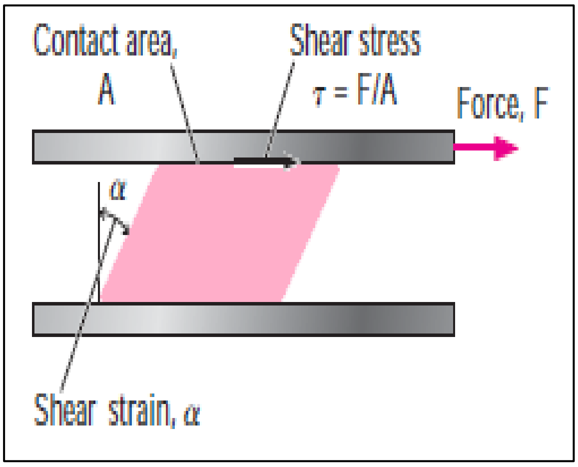


Figure 2. The Fluid

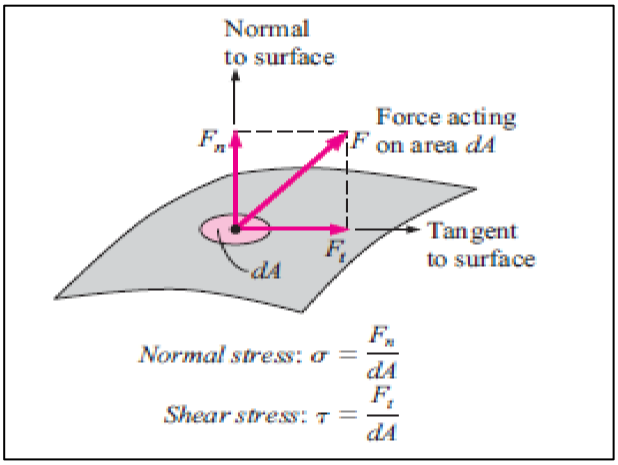


Figure 3. Normal and Shear Stresses

Unlike a liquid, gas does not form a free surface, and it expands to fill the entire available space, as shown in Figure 4.

Also, liquids are almost incompressible, while gases are compressible.

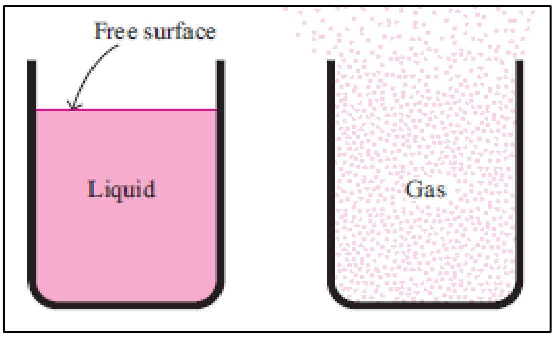


Figure 4. Free Surface for Liquids

1-2 Dimensions

Basic dimensions: mass [M], length [L], and time [T].

Derived dimension: force [F] = mass [M] * acceleration [L/T^2]

All valid equations must satisfy dimensional homogeneity. An equation is dimensionally homogeneous (i.e., all terms have the same dimensions). Table 1 includes the dimensions of some physical quantities.

Table 1. Dimensions

Term	Symbol	Dimensions	
		[M]-[L]-[T]	**[F]-[L]-[T]**
Length	L	L	L
Area	A	L^2	L^2
Volume	V	L^3	L^3
Time	t	T	T
Velocity	v	L/T	L/T
Angular Velocity	ω	T^{-1}	T^{-1}
Acceleration	a	L/T^2	L/T^2
Rate of Discharge	Q	L^3/T	L^3/T
Mass	M	M	FT^2/L
Force	F	ML/T^2	F
Density	ρ	M/L^3	FT^2/L^4
Specific Weight	γ	M/L^2T^2	F/L^3
Dynamic Viscosity	μ	M/LT	FT/L^2
Kinematic Viscosity	ν	L^2/T	L^2/T
Pressure	P	M/LT^2	F/L^2
Momentum (or Impulse)	M (or I)	ML/T	FT
Energy (or Work)	E (or W)	ML^2/T^2	FL
Power	p	ML^2/T^3	FL/T
Surface Tension	σ	M/T^2	F/L

1-3 Units

Table 2. Units Systems

Units System	L	T	M	F
Metric System	cm	s	gm	Dyne
English System	ft	s	lb	pdl
American System	ft	s	slug	lb_f
S. I. System	m	s	kg	Newton

Figure 5 shows the units of force.

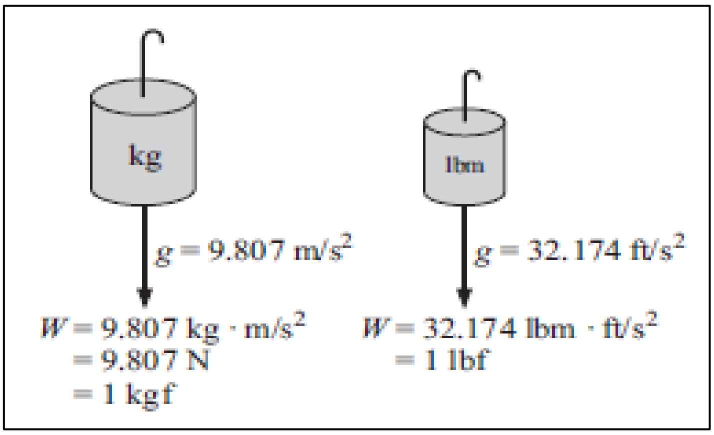


Figure 5. Units of Force

Length: 1 ft = 30.48 cm

1 cm = 0.0328 ft

1 mile = 1609 m

1 mile = 1760 yard

1 yard = 3 ft

1 ft = 12 inch

1 inch = 2.54 cm

Mass: 1 lb = 453.6 gm

1 slug = 32.2 lb

Force: 1 lb_f = 32.2 pdl

1 lb_f = 4.445 N

1 Newton = 10^5 Dyne = 10^5 gm.cm/s^2 = 1 kg.m/s^2

Absolute or scientific (SI) units are Dyne and Newton.

Gravitational units of force = "g" times SI units.

Gravitational or engineering units of force are gm_f and kg_f.

g = 981 cm/s^2 = 9.81 m/s^2 = 32.2 ft/s^2

1 gm_f = 981 Dyne & 1 kg_f = 9.81 Newton

1 Newton = 10^5 dyne & 1 dyne = 1 gm.cm/s^2

Pressure: 1 bar = 10^5 Pa = 10^5 N/m^2

1 bar = 14.7 psi (lb/in^2)

1 bar = 1.02 kg/cm^2

Power: 1 hp = 745.7 W

1-4 Fluid Properties

Density (ρ) It is the mass per unit volume.

ρ = **M / V**

ρ for water = 1 gm/cm^3 = 1000 kg/m^3 & ρ for air = 1.205 kg/m^3

Specific Weight (γ) It is the weight per unit volume.

$$\gamma = \frac{W}{V} = \frac{Mg}{V} = \rho\, g$$

γ for water = 1 gm/cm^3 = 1 t/m^3 = 1000 kg/m^3

γ for water = 62.4 lb/ft^3 = 981 dyne/cm^3 = 9810 N/m^3

γ for mercury =13,546 kg/m^3

γ for air = 1.23 kg/m^3

Specific Volume (S.V.) It is the volume per unit mass.

$$S.V. = \frac{V}{M} = \frac{1}{\rho}$$

Specific Gravity (S.G.) It is the ratio of the density of a fluid to the density of the water (or the air).

$$S.G. = \frac{\rho\ \text{fluid}}{\rho\ \text{water}} = (\text{mathematically})\ \frac{\gamma\ \text{fluid}}{\gamma\ \text{water}}$$

S.G. of some substances at 0°C are tabulated in Table 3.

Table 3. S.G.

Substance	S.G.
Water	1.0
Seawater	1.025
Ice	0.92
Mercury	13.6
Air	0.0013
Wood	0.3 – 0.9
Gasoline	0.7

Viscosity

It is the resistance of motion or translation of one layer of the fluid relative to the other, as shown in Figure 6. It is due to cohesion between the particles of a moving fluid.

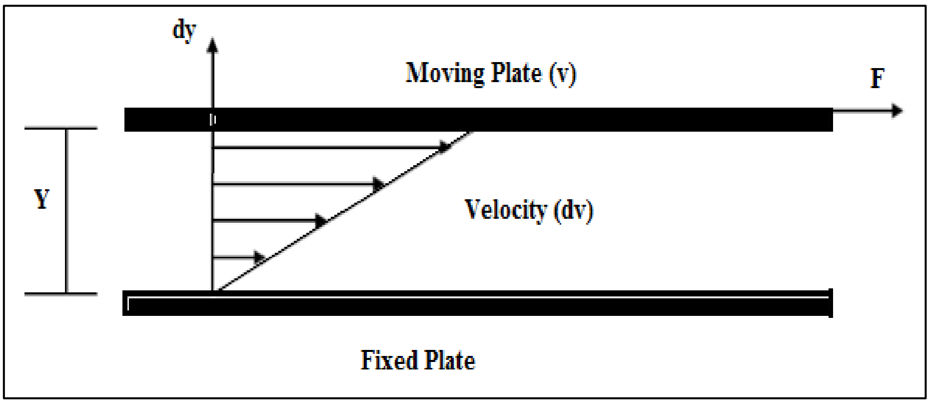


Figure 6. Viscosity

Newton's law of viscosity: "The shear stress on a layer of a fluid is directly proportional to the rate of shear strain".

$$\tau \propto \frac{dv}{dy} = \mu \frac{dv}{dy}$$

τ : Shear stress.

μ : Viscosity or dynamic viscosity.

dv/dy : Shear strain = rate of angular deformation & *Unit:* rad/s

μ = (F/A) / (v/L) = (F/A) / (1/T) = (F * T) / A = (MLT^{-2} * T) / L^2 = M / L T

Poise = Dyne.s/cm^2 = 0.1 kg/m.s

1 Poise = 10^{-5} N.s/cm^2 = 0.1 N.s/m^2 = 0.1 Pa.s

μ for water = 1.14 * 10^{-3} kg/m.s

μ for mercury =1.552 * 10^{-3} kg/m.s

μ for air = 1.78 * 10^{-5} kg/m.s

A fluid moving relative to a body exerts a drag force on the body, partly because of friction caused by viscosity, as shown in Figure 7.

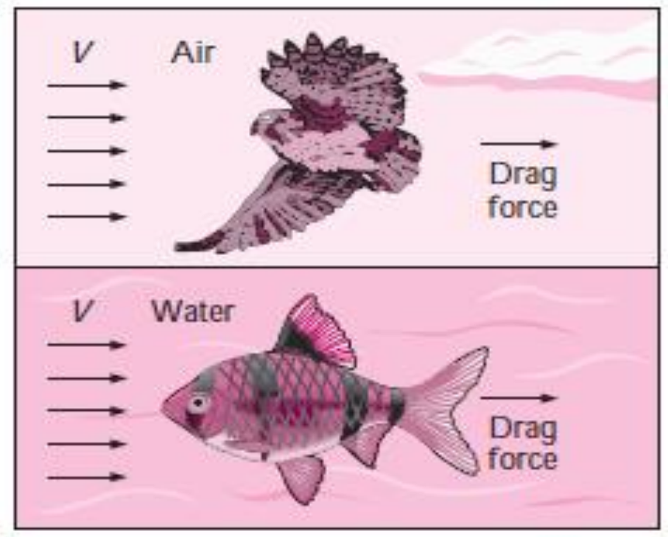


Figure 7. Drag Force

The viscosity of liquids decreases and the viscosity of gases increases with temperature, as shown in Figure 8.

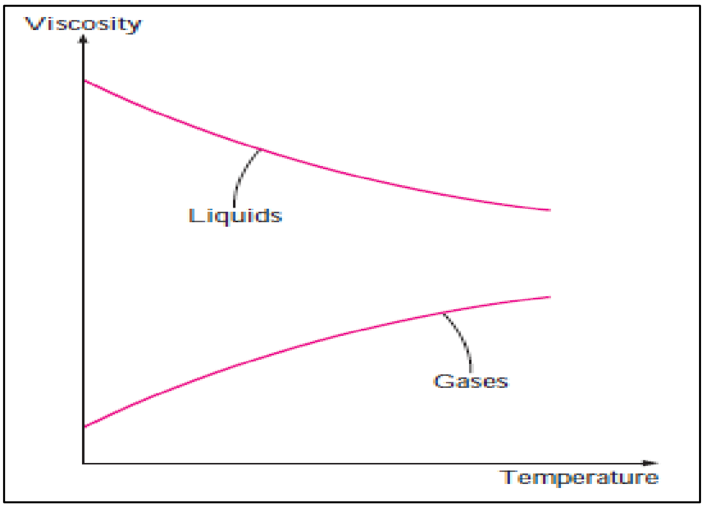


Figure 8. Viscosity versus Temperature

A plot of shear stress versus the rate of deformation (velocity gradient) for a Newtonian fluid is a straight line whose slope is the viscosity of the fluid, as shown in Figure 9. Note that viscosity is independent of the rate of deformation.

The rate of deformation (velocity gradient) of a Newtonian fluid is proportional to shear stress, and the constant of proportionality is the viscosity.

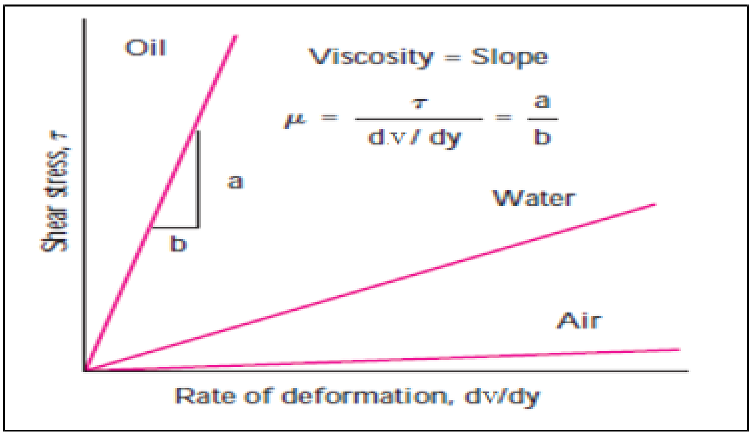
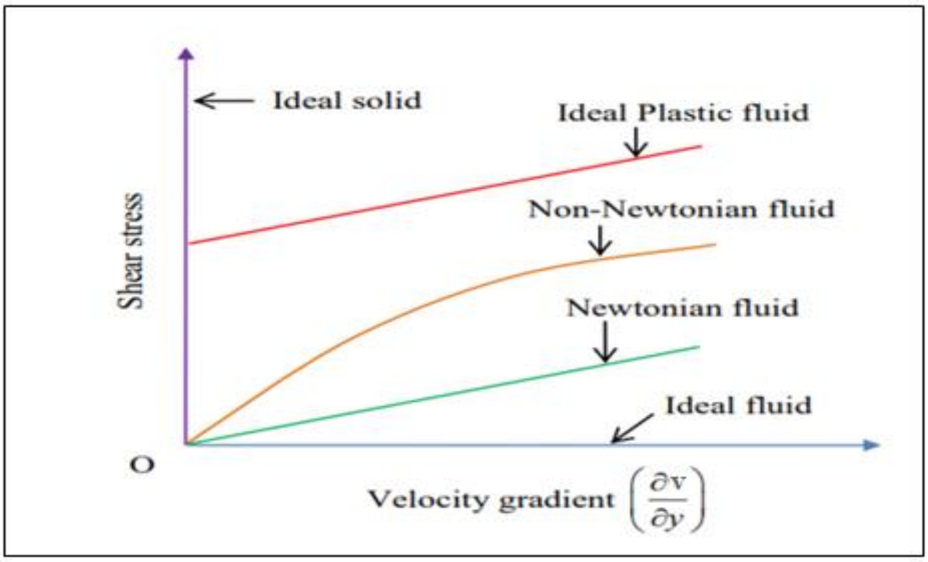


Figure 9. Newtonian Fluids

Figure 10 shows Newtonian and non-Newtonian fluids.



https://tutorialstipscivil.com/civil-topics/types-of-fluid/

Figure 10. Newtonian and Non-Newtonian Fluids

Table 4 includes the dynamic viscosity of some fluids at 1 atm and 20°C.

Table 4. Dynamic Viscosity

Fluid		Dynamic Viscosity (μ), kg/m.s
Water	0°C	0.0018
	20°C	0.0010
	100°C (liquid)	0.00028
	100°C (vapor)	0.000012
Air		0.000018
Mercury		0.0015

Kinematic Viscosity (ν)

$$\nu = \mu / \rho$$

Stoke = $cm^2/s = 10^{-4}\ m^2/s$

ν for water = $1.14 * 10^{-6}\ m^2/s$

ν for mercury = $1.145 * 10^{-4}\ m^2/s$

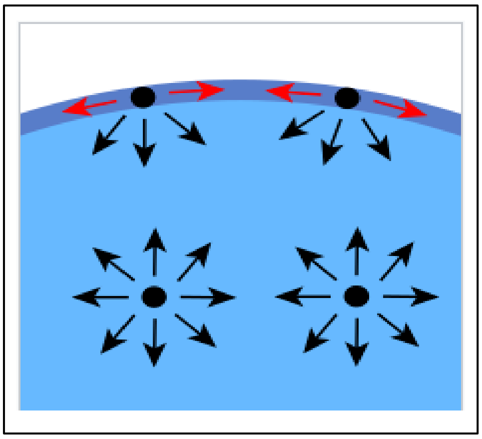
ν for air = $1.46 * 10^{-5}\ m^2/s$

Surface Tension (σ)

It is the tendency of liquid surfaces at rest to shrink into the minimum surface area possible.

As shown in Figure 11, the attractive forces applied to the interior molecule by the surrounding molecules balance each other because of symmetry.

However the attractive forces acting on the surface molecule are not symmetric, and the attractive forces applied by the gas molecules above are usually very small. Therefore, there is a net attractive force acting on the molecule at the surface of the liquid, which tends to pull the molecules on the surface toward the interior of the liquid. This force is balanced by the repulsive forces from the molecules below the surface that are being compressed.



https://en.wikipedia.org/wiki/Surface_tension

Figure 11. Attractive Forces Acting on Liquid Molecules

The resulting compression effect causes the liquid to minimize its surface area. This is the reason for the tendency of the liquid droplets to attain a spherical shape, which has the minimum surface area for a given volume.

The magnitude of this force per unit length is called surface tension σ_s and is usually expressed in the unit N/m (or lbf/ft in English units).

Table 5 includes values of surface tension of some fluids in the air at 1 atm and 20°C.

Table 5. Surface Tension

Fluid		Surface Tension (μ), N/m
Water	0°C	0.076
	20°C	0.073
	100°C	0.059
	300°C	0.014
Mercury		0.440

Another interesting consequence of surface tension is the capillary effect which is the rise or fall of a liquid in a small-diameter tube inserted into the liquid.

The capillary effect is also partially responsible for the rise of water to the top of tall trees.

The curved free surface of a liquid in a capillary tube is called the meniscus.

The strength of the capillary effect is quantified by the contact (or wetting) angle φ, defined as the angle that the tangent to the liquid surface makes with the solid surface at the point of contact, as shown in Figure 12. The surface tension force acts along this tangent line toward the solid surface. A liquid is said to wet the surface when $\varphi < 90°$ and not to wet the surface when $\varphi > 90°$.

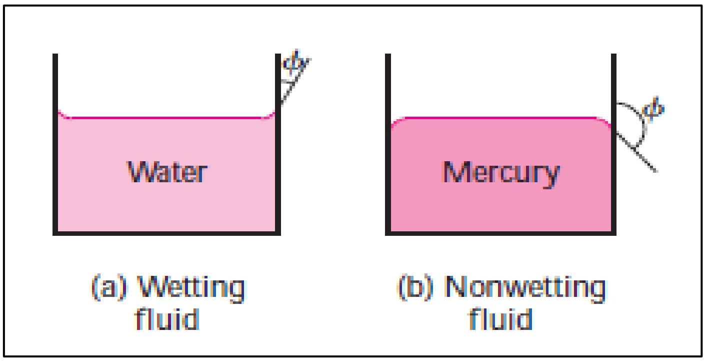


Figure 12. Contact Angles of Fluids

Figure 13 shows the capillary rise of water and the capillary fall of mercury in a small-diameter glass tube.

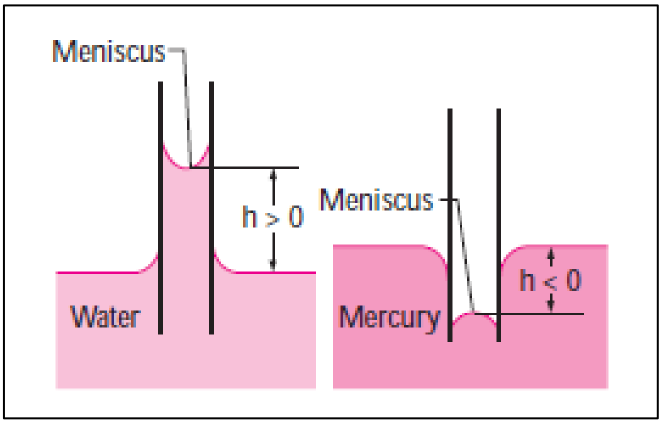


Figure 13. Capillary Effect

Figure 14 shows the forces acting on a liquid column that has risen in a tube due to the capillary effect.

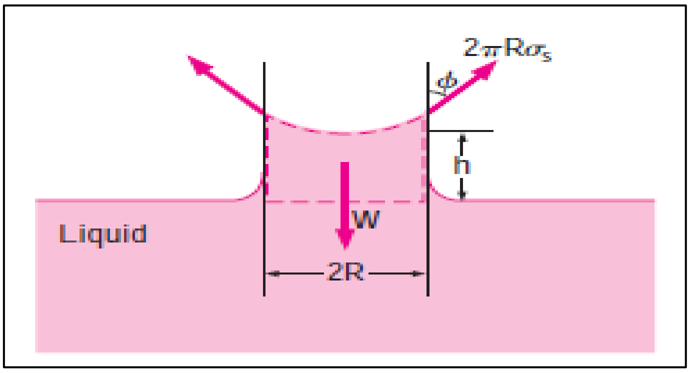


Figure 14. Forces Due to Capillary Effect

Example 1

When 5.6 m^3 of oil weighs 46,800 N, find its density and its specific gravity.

Solution

W = M * g

M = 46,800 / 9.81 = 4770.6 kg

$\therefore$ ρ = M / V = 4770.6 / 5.6 = 852 kg/m^3

S.G. = ρ (oil) / ρ (water)

ρ (water) = 1000 kg/m^3

$\therefore$ S.G. = 852 / 1000 = 0.852

Example 2

When the viscosity of water is 0.00995 Poise, find its value in Pa.s.

Determine the kinematic viscosity in Stoke.

Solution

$\mu = 0.00995 * 0.1 = 9.95 * 10^{-4}$ N.s/m^2

$\therefore\ \mu = 9.95 * 10^{-4}$ Pa.s

$\nu = \mu / \rho$

$\mu = 9.95 * 10^{-4}$ kg/m.s

$\rho = 1000$ kg/m^3

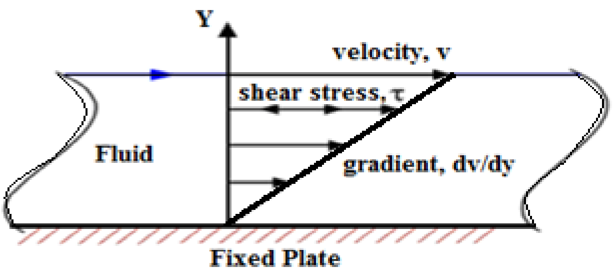
$\nu = (9.95 * 10^{-4}) / 1000 = 9.95 * 10^{-7}$ m^2/s

$\therefore\ \nu = (9.95 * 10^{-7}) * 10^4 = 9.95 * 10^{-3}$ Stoke

Example 3

The velocity is 1.125 m/s at 75 mm from the boundary. The fluid has a viscosity of 0.048 Pa.s and a relative density of 0.913.

Assuming a linear velocity distribution, what are the velocity gradient and the shear stress? Determine the kinematic viscosity.



Solution

$$\frac{dv}{dy} = \frac{1.125}{0.075} = 15\ s^{-1}$$

$\tau = \mu\ (dv/dy) = 0.048 * 15 = 0.72$ Pa

$\nu = \mu / \rho$

$\rho = 0.913 * 1000 = 913$ kg/m^3 (*SI system*)

$\therefore\ \nu = 0.048 / 913 = 5.257*10^{-5}$ m^2/s

Example 4

Bernoulli's equation for ideal fluid flow may be written as:

$(P/\gamma) + z + (v^2/2g) = \text{constant}$

Where: p = pressure, γ = specific weight, z = elevation, v = velocity, g = gravitational acceleration.

Show that the equation is dimensionally homogeneous.

Solution

$[P/\gamma] = [ML^{-1}T^{-2} / ML^{-2}T^{-2}] = [L]$

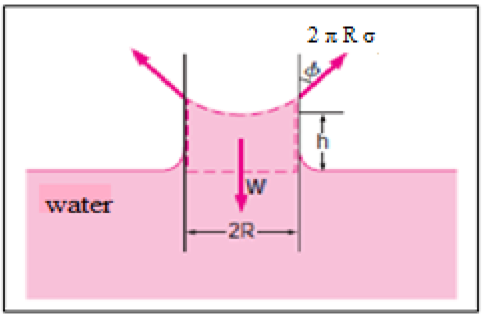
$[z] = [L]$

$[v^2/2g] = [(LT^{-1})^2 / LT^{-2}] = [L]$

∴ The equation is dimensionally homogeneous with dimensions [L]

Example 5

Find the water height at 20°C in a 1.6 mm tube, as shown in the figure.



Solution

$F\sigma = Fw$

$2\,\pi\,R\,\sigma\cos\theta = \gamma\,V$

θ is very small, $\cos\theta = 1$

$V = h * A$

$2\,\pi\,R\,\sigma = \gamma * h * A$

$2\,\pi\,R\,\sigma = \gamma * h * (\pi\,R^2)$

$$h = \frac{2\,\sigma}{\gamma\,R} = \frac{2 * 0.073}{9810 * 0.8 * 10^{-3}} = 0.0186\,m = 18.6\,mm$$

Chapter 2

STATICS OF FLUIDS

1. Fluid Pressure	*2. Measuring Fluid Pressure*
3. Total Pressure on Inclined Submerged Surfaces	*4. Centre of Pressure*
5. Total Pressure on Curved Surfaces	*6. Buoyancy*

2-1 Fluid Pressure

Pressure is the force per unit perpendicular area.

P = F / A

$1 \text{ bar} = 10^5 \text{ Pa} = 10^5 \text{ N/m}^2$

$1 \text{ bar} = 14.7 \text{ psi (lb/in}^2)$

$1 \text{ bar} = 1.02 \text{ kg/cm}^2$

$1 \text{ atm} = 1.103235 * 10^5 \text{ N/m}^2$

Pressure can also be expressed as the height of an equivalent liquid column.

$\mathbf{P = \gamma\, h}$

If P = 2 kg/cm^2, then P = 2000 gm/cm^2 = γ h = 1 * h

$\therefore$ h = 2000 cm = 20 m of water column.

If P = 20 kN/m^2, then P = 20000 N/m^2 = γ h = 9810 * h

$\therefore$ h = 20000 / 9810 = 2.04 m of water column.

Pascal's Laws:

"The intensity of pressure is the same in all directions at any point in a fluid at rest".

"The pressure is the same at any two points in the same elevation in a continuous mass of a fluid at rest".

Atmospheric Pressure: It is the pressure exerted by a column of air of 1 cm^2 cross-sectional area, and a height equal to that of the atmosphere at sea level.

At the sea level, $P_{atm} = 1.03\ kg/cm^2$

$= 10.3$ m of water

$= 76$ cm of mercury.

Gauge Pressure: It is the measured pressure, taking P_{atm} as a datum.

Absolute Pressure: It is the algebraic sum of the atmospheric pressure and the gauge pressure, as shown in Figure 1.

$$P_{abs} = P_{atm} + P_g$$

$P_{gauge} = P_{abs} - P_{atm}$ & $P_{vac} = P_{atm} - P_{abs}$

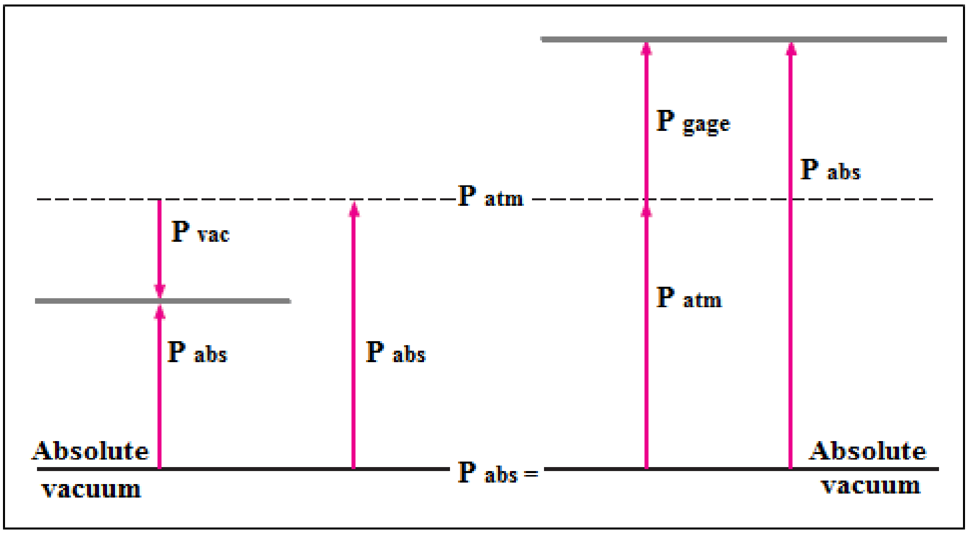


Figure 1. Absolute, Gauge, and Atmospheric Pressures

2-2 Measuring Fluid Pressure

A) *Mechanical Gauges*

Dead Weight Pressure Gauges

It consists of a piston on a cylinder with a known area, and it is connected to the gauge point (in a pipe containing a pressed fluid for example) by a tube, as shown in Fig. 2.

The pressure P:

P = Weight / Area of Piston

This gauge is suitable for measuring very high pressures.

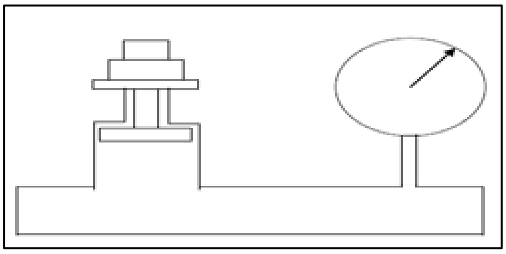


Figure 2. Dead Weight Pressure Gauges

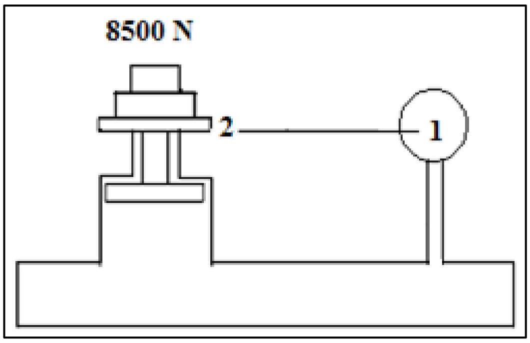
Example 1

A dead weight pressure gauge is used to measure the pressure of a liquid in a pipe at the same level. The value of the weight is 8500 N.

1) If the area of the piston is 100 cm^2, determine the pressure in the pipe.

2) If the piston is 0.5 m below the pipe, determine the pressure in the pipe.

Solution

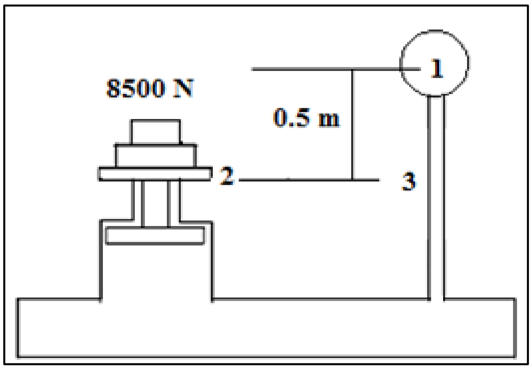


1. At the same level:

$P_1 = P_2$

$\therefore P_1 = W/A = 8500/100 = 85\ N/cm^2$

2. <u>At different levels:</u>



$P_3 = P_1 + \gamma h = P_2$

$\therefore P_1 = P_2 - \gamma h$

$= 85 - (981 \text{x} 10^{-5} * 50)$

$= 84.51 \text{ N/cm}^2$

Figure 3 shows how to lift a large weight by a small force by the application of Pascal's law.

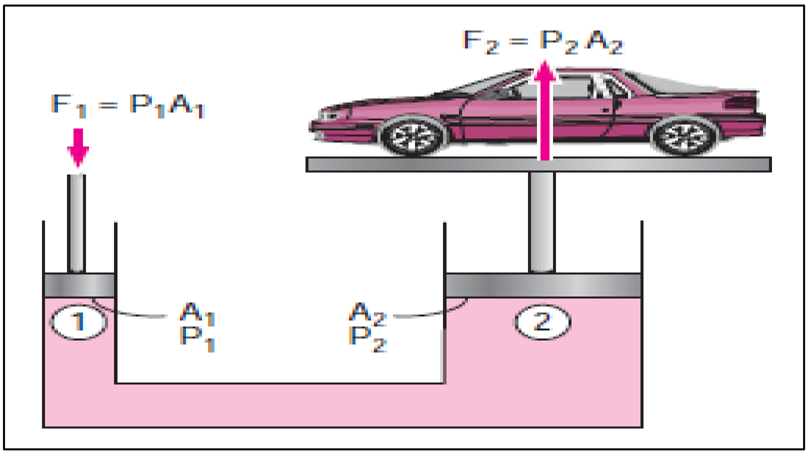


Figure 3. Application of Pascal's Law

B) Tube Gauges

I- Piezometer Tube

It is an open tube, in which one end is open to the atmosphere, and the other is attached to a pipe containing a pressed liquid, as shown in Figure 4. The height of the liquid in the tube gives the head pressure.

It is used for measuring moderate pressures.

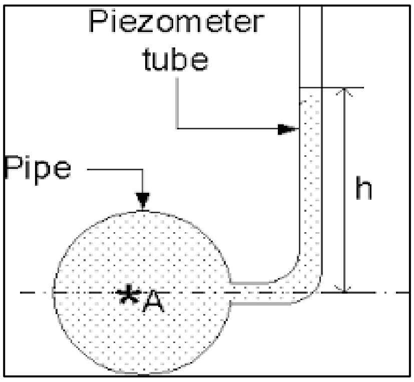


Figure 4. Piezometer

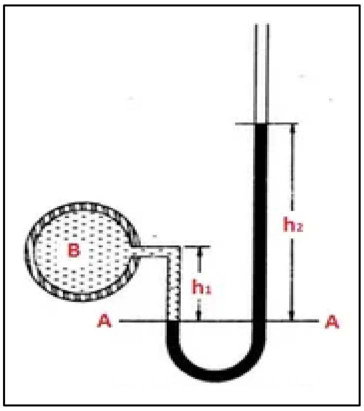
https://www.quora.com/What-are-the-uses-of-piezometers

The piezometer tube is not suitable for measuring negative pressure; otherwise, the air will enter the pipe through the tube.

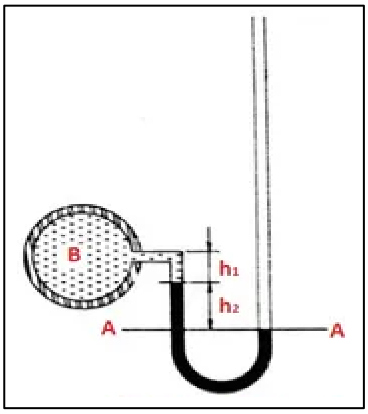
II- Manometer

It is a U-shaped tube with a liquid (mercury in general), as shown in Figure 5. One end of the tube is open to the atmosphere, and the other is attached to the gauge point. It is suitable for measuring both high and negative pressures.

Differential manometers measure the pressure difference, as shown in Figures 6 and 7.

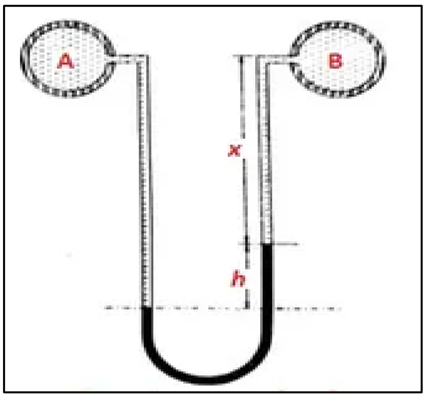


Positive Pressure



Negative Pressure

Figure 5. Simple Manometers
https://www.theengineerspost.com/types-of-manometers/



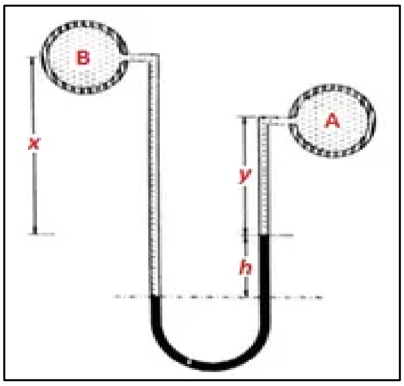


Figure 6. Differential Manometers
https://www.theengineerspost.com/types-of-manometers/

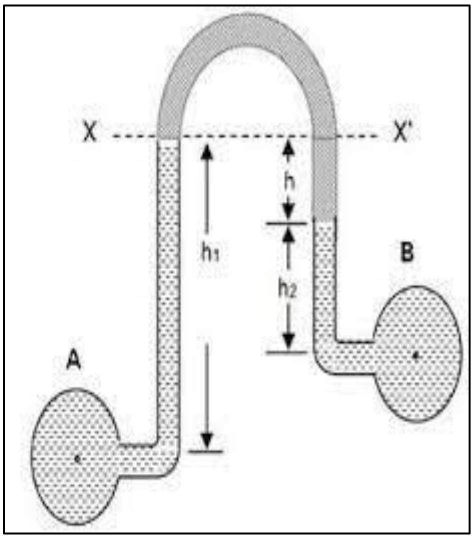
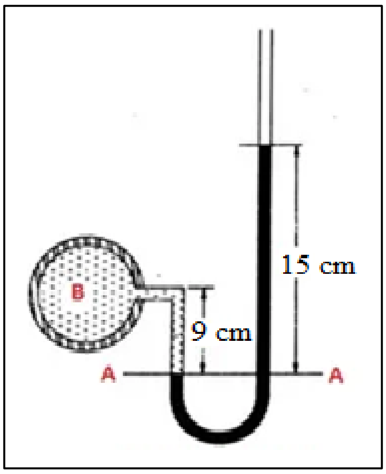


Fig. 7 Inverted Differential Manometer

https://www.vssut.ac.in/lecture_notes/lecture1525499900.pdf

Example 2

A simple manometer with mercury is used for measuring the pressure of oil in a pipeline, as shown in the Figure. The specific gravity is 0.8 for oil and 13.6 for mercury. Determine the absolute pressure of oil in the pipe in kg/cm^2.



Solution

At the datum, $P_L = P_R$

$P + (\gamma_o * 6) = (\gamma_m * 15)$

$\gamma_o = SG_o * \gamma_w = 0.8 * 0.001 = 0.0008 \text{ kg/cm}^3$

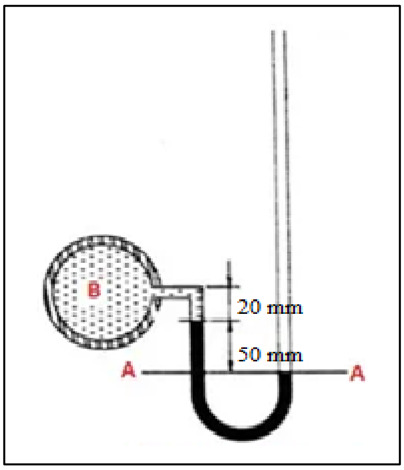
$\gamma_m = SG_m * \gamma_w = 13.6 * 0.001 = 0.0136 \text{ kg/cm}^3$

$P = (\gamma_m * 15) - (\gamma_o * 6) = 0.2 \text{ kg/cm}^2$

$P_{abs} = P_{atm} + P_g$

$\therefore P_{abs} = 1.03 + 0.2 = 1.23 \text{ kg/cm}^2$

Example 3



A simple manometer with mercury is used for measuring the pressure of water in a pipeline. Mercury is 13.6 specific gravity.

Determine the pressure in the pipe.

Solution

At the datum, $P_L = P_R$

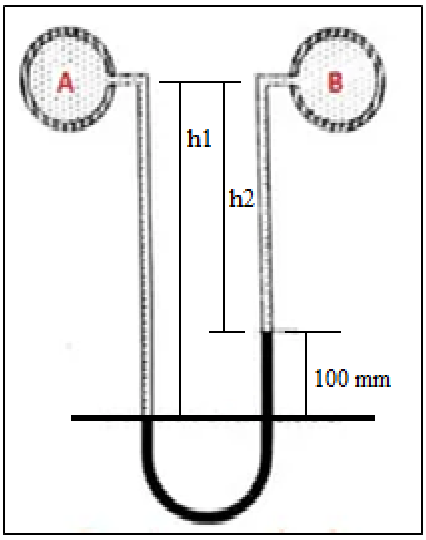
$P + (\gamma_w * 20) + (\gamma_m * 50) = 0$

$\therefore\ h = -20 - (13.6 * 50) = -700$ mm of water

Example 4

A differential manometer is connected between two points A and B in a pipe of oil (specific gravity = 0.8). The reading (difference in mercury levels) is 100 mm.

Determine the difference in pressures between A and B in terms of head of water and gm/cm^2.



Solution

At the datum, $P_L = P_R$

$P_A + (\gamma_o * h_1) = P_B + (\gamma_o * h_2) + (\gamma_m * 100)$

$P_A - P_B \quad = \gamma_o * (h_2 - h_1) + \gamma_m * (h_1 - h_2)$

$\quad = (h_1 - h_2) * (\gamma_m - \gamma_o)$

$\therefore\ h_A - h_B = h\,(S.G_m - S.G_o) = 10\,(13.6 - 0.8)$

$\quad = 128$ cm of water

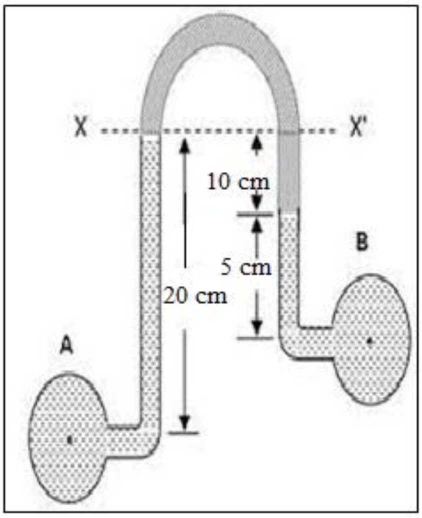
& $P_A - P_B = \gamma_w * h = 1 * 128 = 128$ gm/cm^2

Example 5

An inverted differential manometer with oil of a specific gravity of 0.75 is connected to two pipes of water A&B. The pressure at A is 1.5 m of water.

Determine the pressure in pipe B.

Solution



At the datum, $P_L = P_R$

$P_A - (\gamma_W * 20) = P_B - (\gamma_W * 5) - (\gamma_O * 10)$

$(P_A / \gamma_W - P_B / \gamma_W) = (1*20) - (1*5) - (0.75*10)$

$P_B / \gamma_W = 150 - 20 + 5 + 7.5 = 142.5$ cm of water

$\therefore\ P_B = \gamma_W * h = 1 * 142.5 = 142.5\ gm/cm^2$

2-3 Total Pressure on Inclined Submerged Plane Surfaces

For a fluid at rest, there are no shear stresses.

The pressure forces from the fluid are normal to the exposed surfaces. The pressure force is equal to the multiplication of the pressure by the area of the surface.

When analyzing hydrostatic forces on submerged surfaces, the atmospheric pressure can be subtracted for simplicity when it acts on both sides of the structure, as shown in Figure 8.

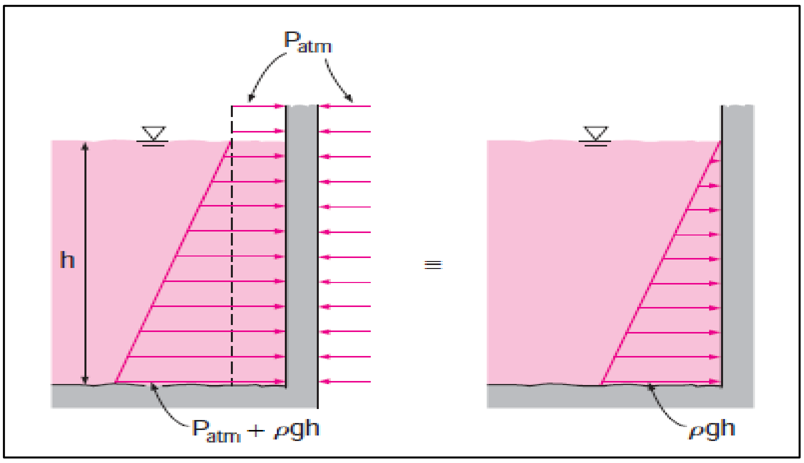


Figure 8. Hydrostatic Forces

For a submerged plane inclined surface in a liquid, as shown in Figure 9,

Pressure on element $P = \gamma\ h = \gamma\ (y \sin\theta)$

Area of the element = dA

Pressure force on the element $dF = P\ dA = \gamma\ y \sin\theta\ dA$

Total pressure force on the surface $F = \int dF$

$$F = \int \gamma\ y \sin\theta\ dA = \gamma \sin\theta \int y\ dA$$

$$\int y\ dA = y'\ A = \text{First moment of area}$$

$$\mathbf{F = \gamma\ y' \sin\theta\ A = \gamma\ h_c\ A = P'\ A}$$

P': Pressure at C (Centroid) of the surface.

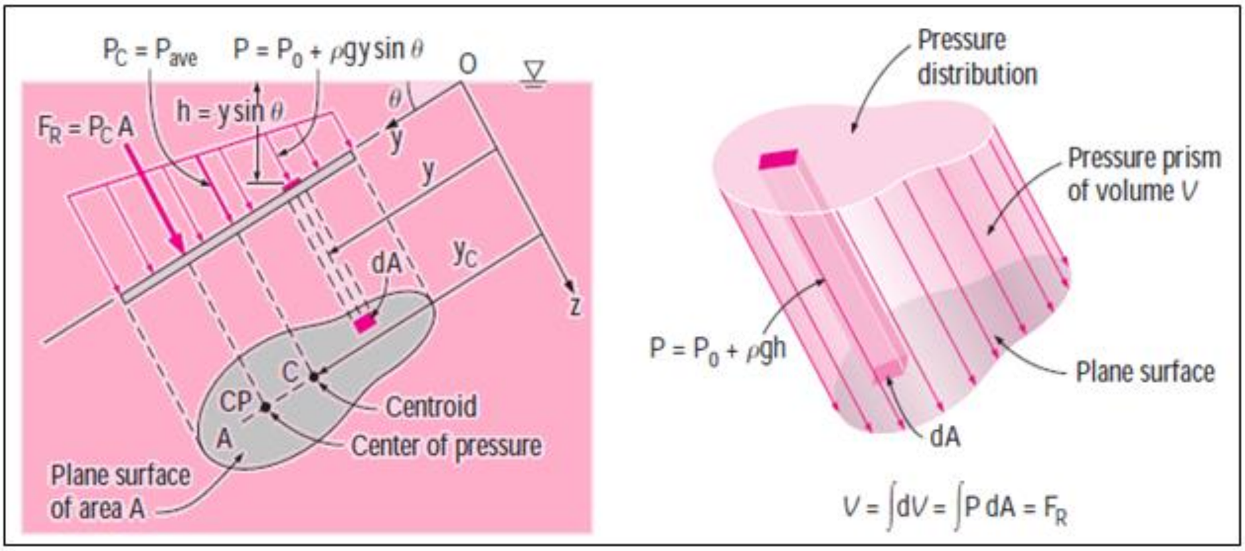


Figure 9. Hydrostatic Force on an Inclined Plane Surface

Where:

θ : Angle between the inclined surface and liquid top surface.

h : Depth of center of gravity (C) of element below the liquid top surface.

h_c : Depth of center of gravity (C) of the surface below the liquid top surface.

y : Distance from O to C of the element.

y_c : Distance from O to C of the surface.

γ : Specific weight of the liquid.

The resultant force acting on a plane surface is equal to the product of the pressure at the centroid of the surface and the surface area, and its line of action passes through the center of pressure, as shown in Figure 10.

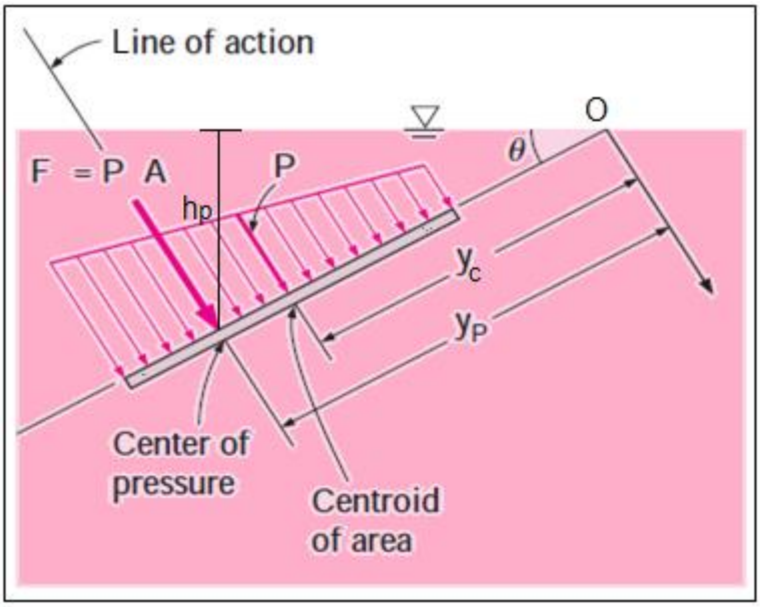
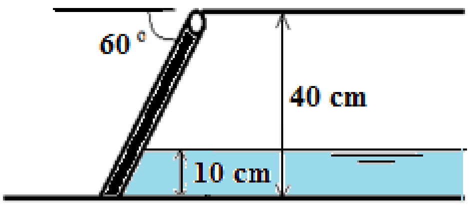


Figure 10. Resultant Hydrostatic Force on an Inclined Plane Surface

Example 6

A tunnel 40 cm * 40 cm is covered at its outlet by a gate that is inclined at 60° with horizontal and is hinged at the upper edge. The depth of water in the tunnel is 10 cm.



Determine the pressure force on the gate.

Solution

$F = \gamma\, h_c\, A$ $\gamma = 1$ gm/cm^3 $h_c = 5$ cm

$A = (10 / \sin 60) * 40 = 461.88$ cm^2 $\therefore F = 1 * 5 * 461.88 = 2309$ gm

Example 7

A rectangular tank 5m long and 2 m wide contains water 2.5 m deep.

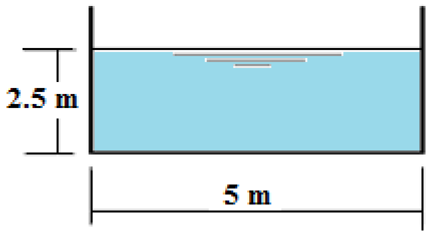
Determine the pressure force on the base of the tank.

Solution *Total Pressure on Horizontal Submerged Surfaces*

$F = \gamma\, h_c\, A$ $\gamma = 1\ t/m^3$

$h_c = 2.5\ m$

$A = 5 * 2 = 10\ m^2$



$\therefore\ F = 1 * 2.5 * 10 = 25\ ton$

Example 8

A circular gate 1.0 m in diameter closes a hole in a vertical retaining wall against seawater that has a specific gravity of 1.03. The center of the hole is 2.0 m below the water's top surface.

Determine the pressure force on the gate.

Solution *Total Pressure on Vertical Submerged Surfaces*

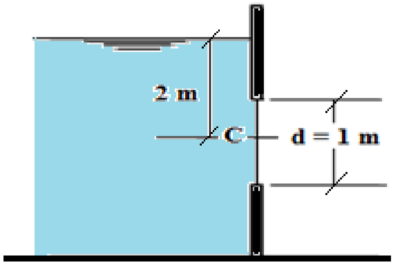
$F = \gamma\, h_c\, A$

$S.G = 1.03 = \gamma / \gamma_w$

$\gamma = 1 * 1.03 = 1.03\ t/m^3$

$h_c = 2\ m$

$A = \pi\, d^2 / 4 = (3.14 * 1^2) / 4 = 0.785\ m^2$



$\therefore\ F = 1.03 * 2 * 0.785 = 1.62\ t$

2-4 Centre of Pressure

For a horizontal submerged surface, the pressure is constant over the surface. Then, the pressure force acts at the center of gravity (C) of the surface, and its line of action is normal to the area of the surface.

For a not horizontal submerged surface, as shown in Figure 11, the pressure is linearly distributed over the surface. Then, the center of pressure C_P is below the center of gravity C of the surface due to the linear variation of pressure with the depth.

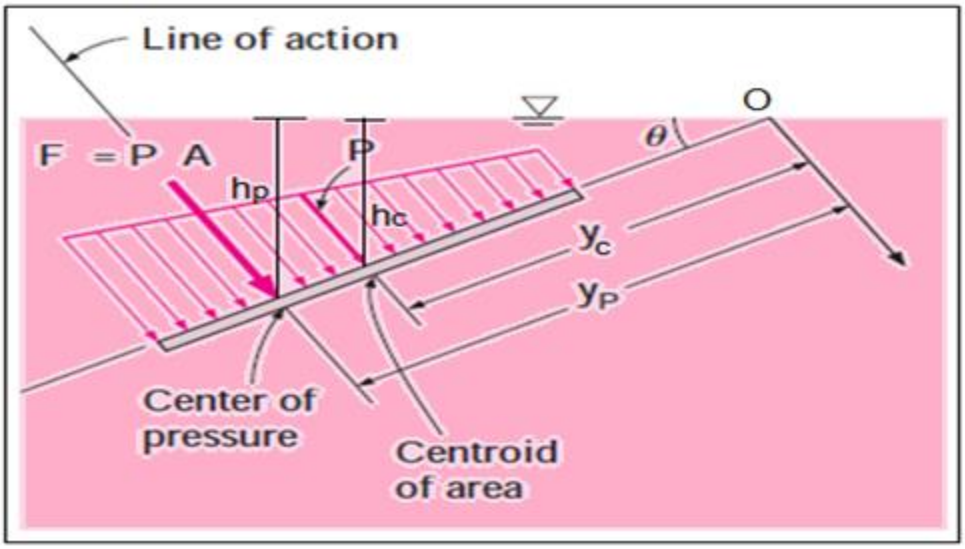


Figure 11. Hydrostatic Force on an Inclined Plane Surface

Taking moments of the pressure forces of element about O:

$M = \int dF\, y = \int (\gamma\, h\, dA)\, y = \int (\gamma\, y \sin\theta\, dA)\, y = \int \gamma\, y^2 \sin\theta\, dA = \gamma \sin\theta \int y^2\, dA$

$\int y^2\, dA = 2^{nd}$ moment of area = I_o = Moment of inertia of the surface about O

$M = \gamma \sin\theta\, I_o$ .... (1)

Taking a moment of the pressure force of the surface about O:

$M = F\, y_P = F\, (h_P / \sin\theta)$ .... (2) $(\sin\theta = h_P / y_P)$

From (1) and (2),

$\gamma \sin\theta\, I_o = F\,(h_p / \sin\theta)$

$F = \gamma \sin\theta\, I_o / (h_p / \sin\theta) = \gamma \sin^2\theta\, I_o / h_p$

$h_p = (\gamma \sin^2\theta\, I_o) / F = (\gamma \sin^2\theta\, I_o) / \gamma\, h_c\, A$

$h_p = (I_o \sin^2\theta) / (h_c\, A)$ .... (3)

But $I_o = I_c + (A\, y_c^2)$

I_c: 2nd moment of area about the center of gravity (C).

Then, in Eq. (3),

$h_p = \{(I_c + A\, y_c^2) \sin^2\theta\} / h_c\, A$

$y_c \sin\theta = h_c$

$\therefore y_c = h_c / \sin\theta$

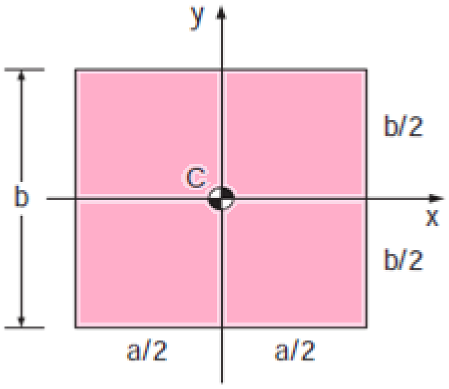
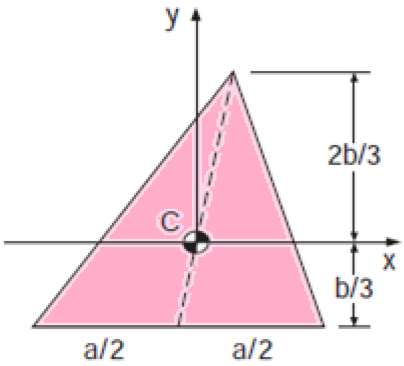
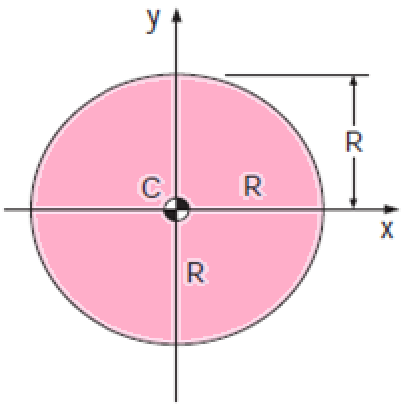
$$h_p = \frac{I_c \sin^2\theta + A\, y_c^2 \sin^2\theta}{h_c\, A} = \frac{I_c \sin^2\theta + A \sin^2\theta\,(h_c^2 / \sin^2\theta)}{h_c\, A} = \frac{I_c \sin^2\theta + A\, h_c^2}{h_c\, A}$$

$$\therefore \boldsymbol{h_p = h_c + \frac{I_c \sin^2\theta}{h_c\, A}}$$

The metacenter is the distance between the center of pressure and the center of gravity. It is the term $(I_c \sin^2\theta) / h_c\, A$.

The common values for I_c are illustrated in Table 1.

Table 1. Common Values of I_c

Name	Shape	Area	2nd Moment of Area about C (I_c)
Rectangle		(a*b)	(a * b^3) / 12
Triangle		(a*b) / 2	(a * b^3) / 36
Circle		π R^2	π R^4 / 4

The pressure at the centroid of a surface is equivalent to the average pressure on the surface, as shown in Figure 12.

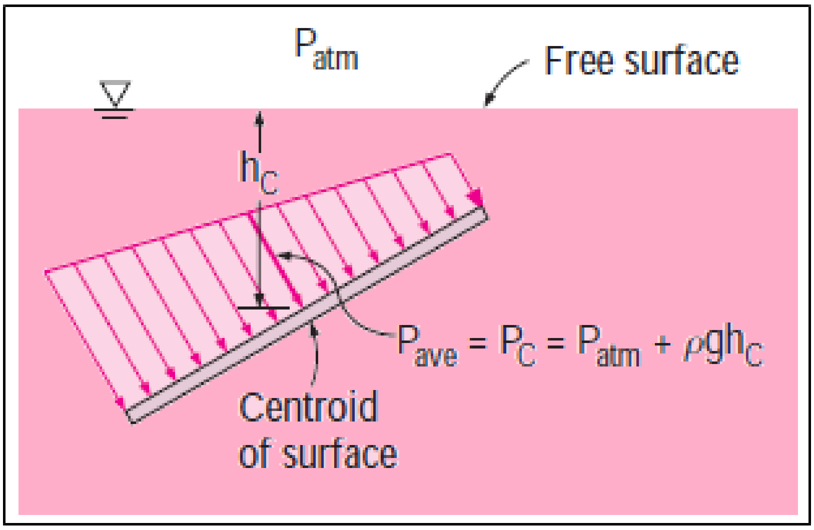


Figure 12. Average Pressure on an Inclined Plane Surface

The resultant force acting on a plane surface is equal to the product of the pressure at the centroid of the surface and the surface area, and its line of action passes through the center of pressure, as shown in Figure 13.

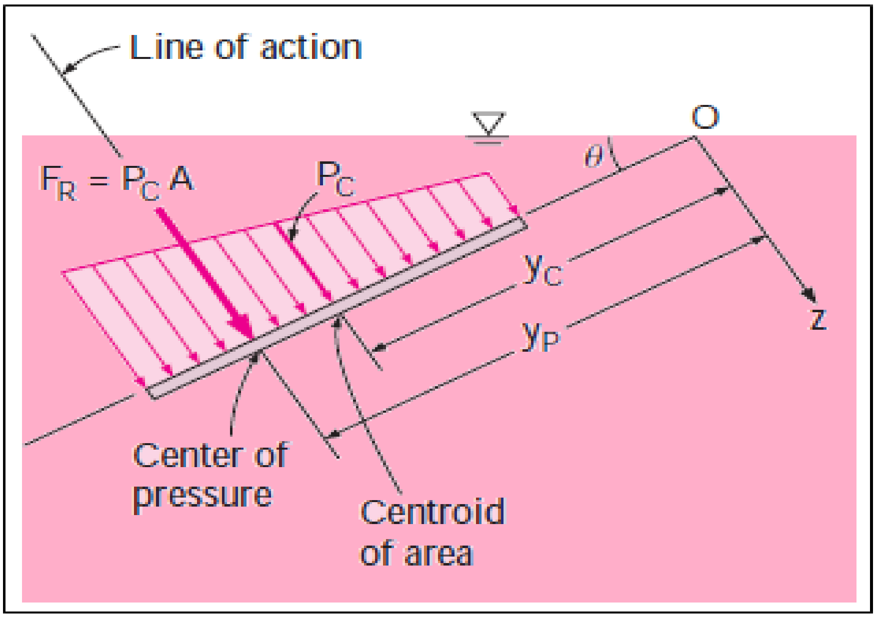


Figure 13. Center of Pressure on an Inclined Plane Surface

Figure 14 illustrates the hydrostatic force acting on the top surface of a submerged rectangular plate for inclined, vertical, and horizontal cases.

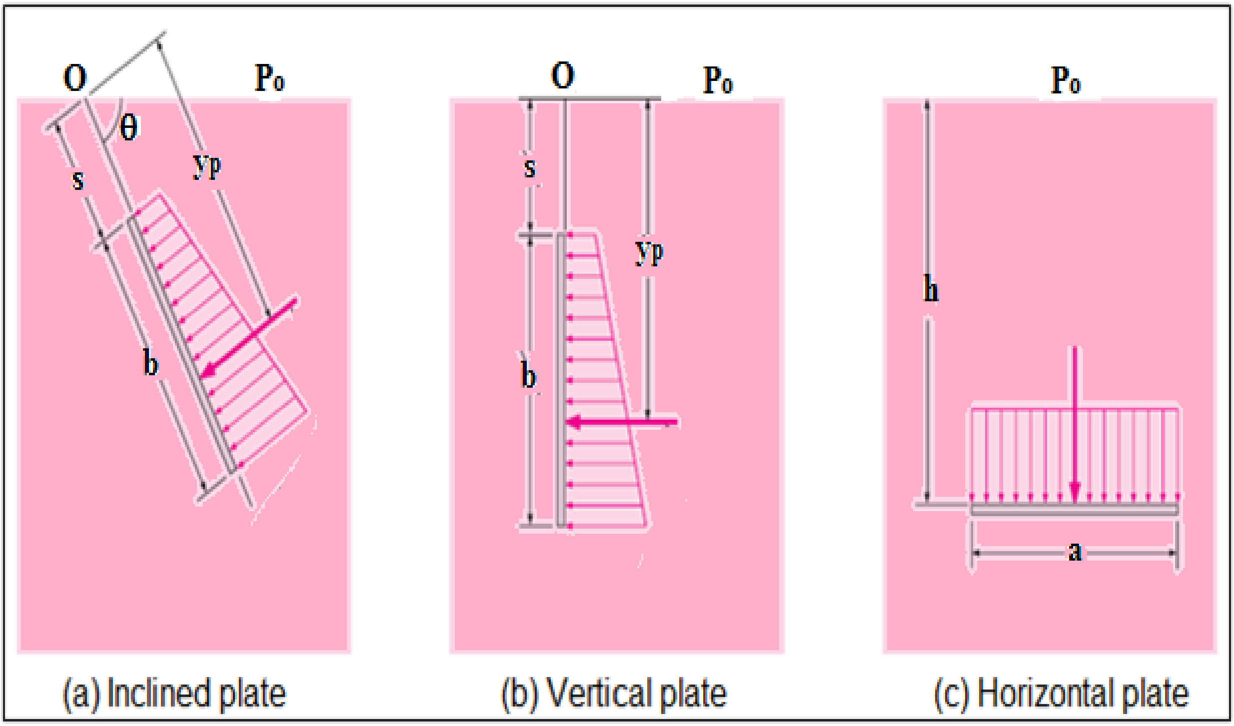


Figure 14. Hydrostatic Force Acting on a Submerged Rectangular Plate

Inclined plate: $F_R = \{P_o + \rho g(s + b/2)\sin\theta\}\ a\ b$

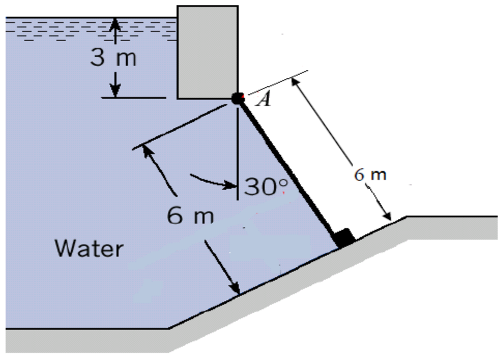
Vertical plate: $F_R = \{P_o + \rho g(s + b/2)\}\ a\ b$

Horizontal plate: $F_R = \{P_o + \rho g h\}\ a\ b$

Example 9

A rectangular gate (6 m * 4 m) is hinged at A and supported by a stopper as shown in the Figure.

Find the reaction at the hinge neglecting the weight of the gate.



Solution

$F = \gamma\, h_c\, A$

$F = \gamma\, \{(3 + 3 \cos 30°)\} * A$

$= 9810 * 5.6 * (6*4)$

$= 1{,}318{,}011 \text{ N} \simeq 1{,}318 \text{ kN}$

$$h_p = \frac{(I_c \sin^2 \theta)}{h_c\, A} + h_c$$

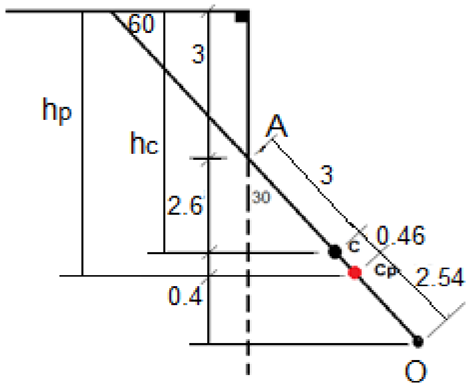
$I_c = b\, h^3 / 36 = (4 * 6^3) / 12 = 72 \text{ m}^4$

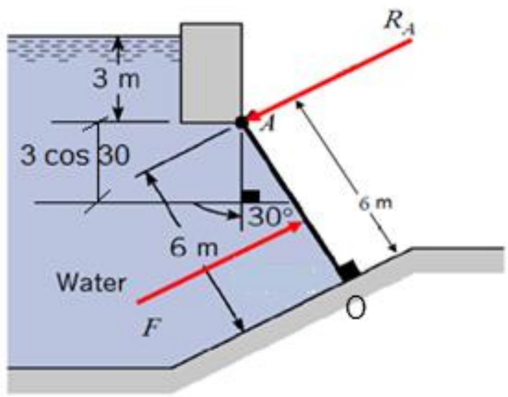
$$h_p = \frac{(72 \sin^2 60°)}{5.598 * 24} + 5.6 = 0.4 + 5.6 = 6 \text{ m}$$

$\sum M = 0$

$6 * R_A = 2.54 * F$

$\therefore R_A = (2.54*1{,}318) / 6 = 557.95 \text{ kN}$





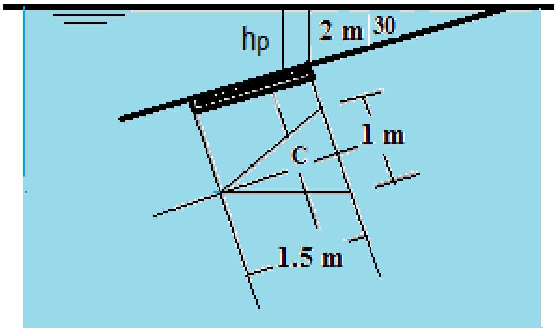
Example 10

A triangular plate of 1.0 m base and 1.5 m height is submerged in water. The plane of the plate is inclined at 30º with the top surface of the water. The base is 2.0 m below the water surface.

1- Determine the total pressure on the plate.

2- Determine the position of the center of pressure (P).

Solution



1- $F = \gamma h_c A$

$\gamma = 1\ t/m^3$

$h_c = 2 + (0.5 * \sin 30) = 2.25\ m$

$A = 0.5 * 1 * 1.5 = 0.75\ m^2$

$\therefore F = 1 * 2.25 * 0.75 = 1.69\ t$

2- $h_p = \dfrac{(I_c \sin^2 \theta)}{h_c A} + h_c$

$I_c = b h^3 / 36 = (1 * 1.5^3) / 36 = 0.094\ m^4$

$h_p = \dfrac{(0.094 \sin^2 30)}{2.25 \times 0.75} + 2.25 = 2.264\ m$

2-5 <u>Total Pressure on Curved Surfaces</u>

When a plane surface is submerged in a liquid, the hydrostatic forces acting perpendicularly on all elements have the same direction forming a system of parallel forces, as shown in Figure 15.

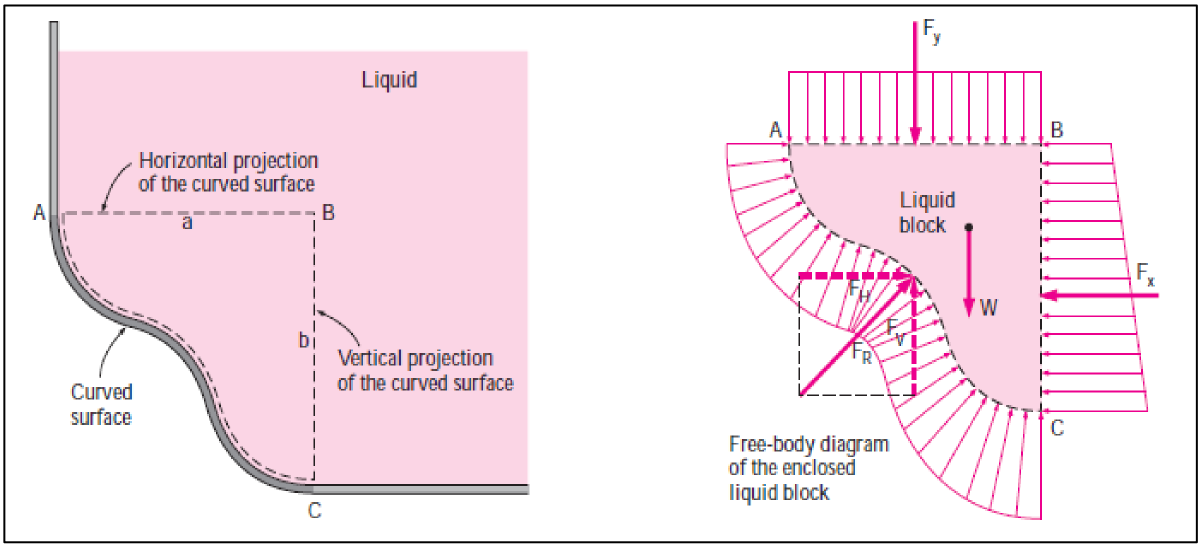


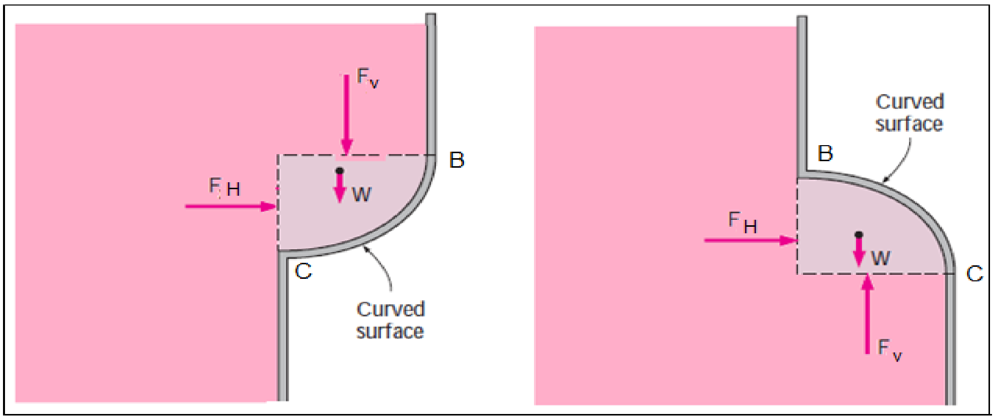
Figure 15. Hydrostatic Force Acting on a Curved Surface

In the case of a curved surface, all elements do not lie in the same plane. So, the hydrostatic forces (although perpendicular to their respective elements) do not form a system of parallel forces.

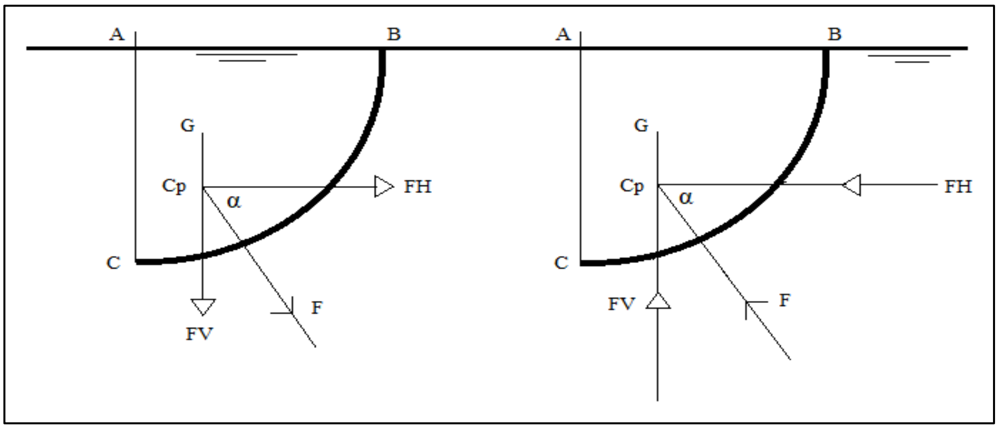
To get the hydrostatic force on a submerged curved surface, both horizontal and vertical components are calculated first. Then, their resultant will be the required force.

The horizontal force F_H, as shown in Figure 16 (A and B), is the total horizontal pressure on the vertical projection of the curved surface BC. It acts at the center of pressure of the vertical projection of the surface.

The vertical force F_V is the total weight of the liquid above the surface, in the portion ABC. It acts downward at the center of gravity of the surface.



(A)



(B)

Figure 16. Forces Acting on a Curved Surface

The magnitude of the total hydrostatic force F is:

$$F = \sqrt{F_H^2 + F_V^2}$$

The direction of the total hydrostatic force F is:

$$\tan \propto = \frac{F_V}{F_H}$$

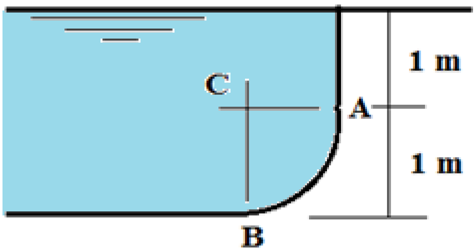
Where α is the angle between the total force F and the horizontal.

Example 11

A curved gate AB, as shown in the figure, is a quadrant of a circular cylinder of radius 1 m.

Determine the total force on the gate per meter length.

Solution



The horizontal force:

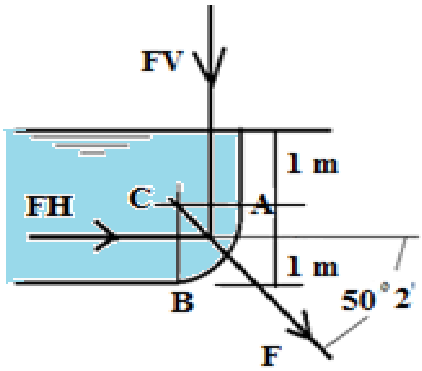
$F_H = \gamma\, h_c\, A$

$\gamma = 1\ t/m^3$

$h_c = 1 + 0.5 = 1.5\ m$

$A = 1 * 1 = 1\ m^2$

$\therefore\ F_H = 1 * 1.5 * 1 = 1.5\ t$



The vertical force:

F_V = Weight of water over curved surface AB

$= \gamma$ * Volume $= \gamma$ * A * Unit length

$\therefore F_V = 1 * [(0.25)\ (\pi\ 2^2/\ 4) + (1 * 1)] * 1 = 1.79\ t$

Magnitude of the total force:

$\therefore F = (F_H^2 + F_V^2)^{1/2} = (1.5^2 + 1.79^2)^{1/2} = 2.34\ t$

Direction of the total force:

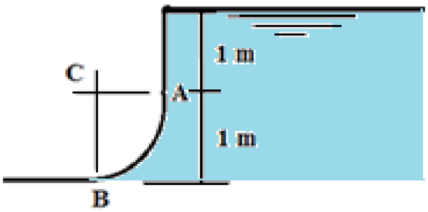
$\tan \alpha = F_V / F_H = 1.79 / 1.5 = 1.19$

$\alpha = \tan^{-1} (1.19) = 50^o\ 2'$

Where α is the angle between the total force F and the horizontal.

Exercise

Determine the total force on the shown gate per meter length.



2-6 Buoyancy

For a submerged body, there is no horizontal component for the hydrostatic force ($F_H = 0$).

The vertical component of the hydrostatic force F_V only exists and is called the buoyant force or upthrust F_B. This force is equal to the difference between the vertical hydrostatic forces acting on both the lower and upper surfaces of the submerged body, as shown in Figure 17.

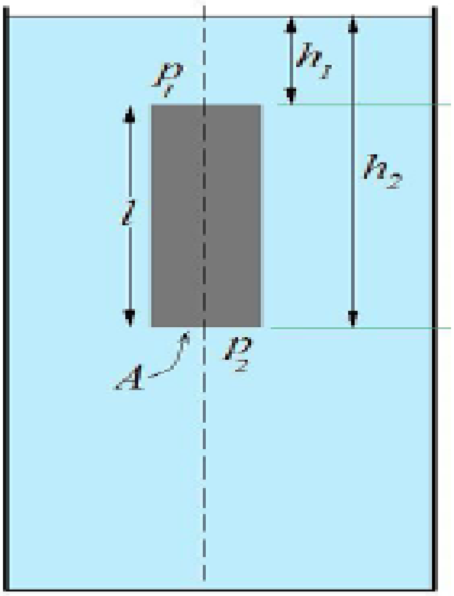


Figure 17. Buoyant Force

$F_B = p_2\ A - p_1\ A = A\ (p_2 - p_1)$

$p_1 = \varrho\ g\ h_1$ & $p_2 = \varrho\ g\ h_2$

$F_B = \varrho\ g\ A\ (h_2 - h_1) = \varrho\ g\ A\ (l)$

$F_B = \varrho\ g\ V$

As shown in Figure 18,

The pressures at the top surface of the plate = ρ_f g s

The pressures at the bottom surface of the plate = ρ_f g (s + h)

The hydrostatic force on the top surface is $F_{top} = \rho_f g s A$ (downward)

The hydrostatic force on the bottom surface is $F_{bottom} = \rho_f g (s + h) A$ (upward)

The difference between these two forces is a net upward force, which is the buoyant force.

$$F_B = F_{bottom} - F_{top} = \rho_f g (s + h) A - \rho_f g s A = \rho_f g h A = \rho_f g V$$

Where ρ_f is the fluid density.

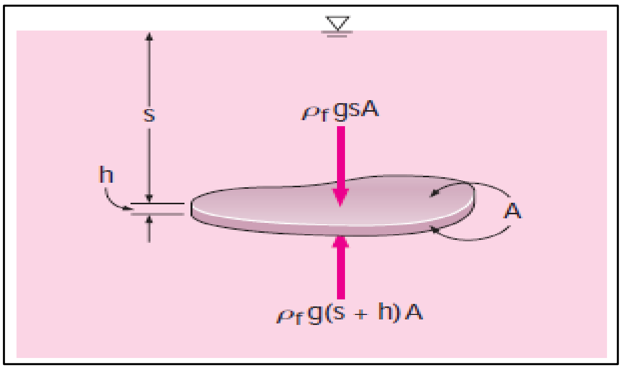


Figure 18. Buoyant Force on a Plate

The center of buoyancy is the center of the area of the submerged section. This is Archimedes' principle.

Thus, the buoyant force depends only on the volume of the body (not its geometry). It is equal to the weight of the displaced liquid and acts vertically upward at the center of buoyancy B, which is the center of gravity of the volume of the displaced liquid.

A solid body dropped into a fluid will sink, float, or remain at rest at any point in the fluid, depending on its density relative to the density of the fluid, as shown in Figure 19.

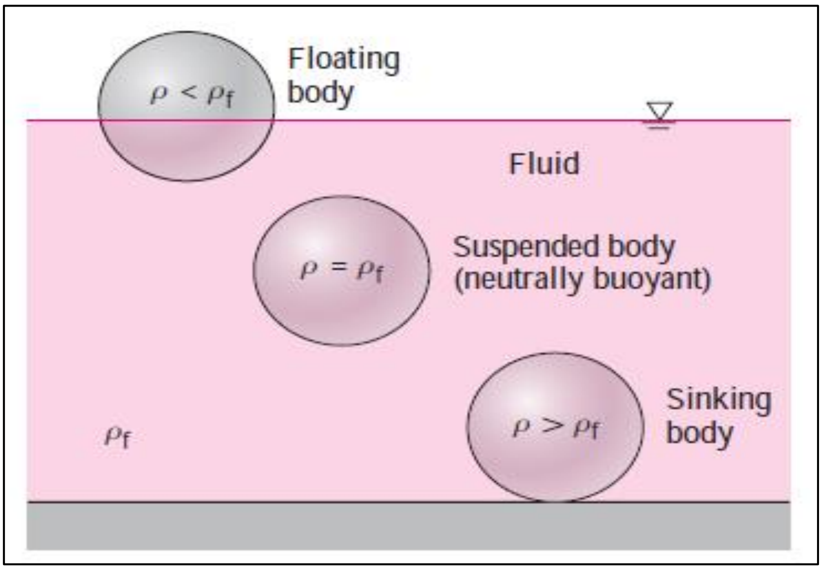


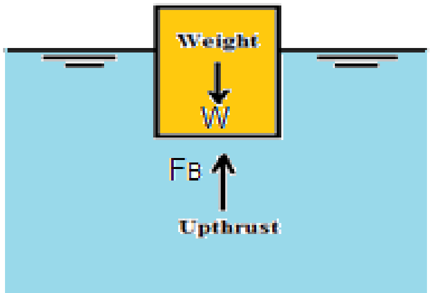
Figure 19. Dropping a Body into a Fluid

<u>In the shown figure,</u>

a) $F_B < W$ the body sinks to the bottom.

b) $F_B = W$ the body floats.

c) $F_B > W$ the body rises to the surface.

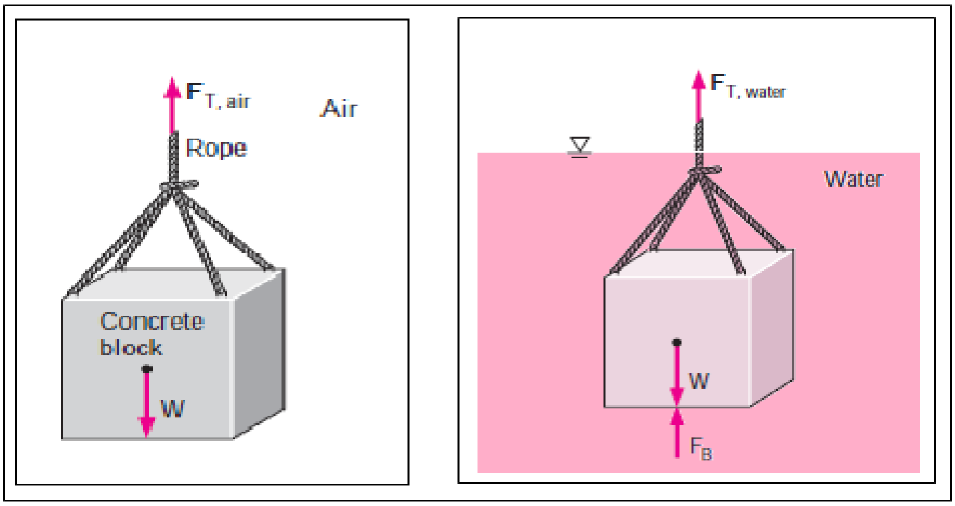


<u>Example 12</u>

A crane is used to lower weights into the sea (density = 1025 kg/m^3) for an underwater construction project, as shown in Figure.

Determine the tension in the wire of the crane due to a rectangular 0.4 m * 0.4 m * 3 m concrete block (density = 2300 kg/m^3) when it is: (*a*) suspended in the air and (*b*) completely immersed in water.

Solution



$V = (0.4\ m)*(0.4\ m)*(3\ m) = 0.48\ m^3$

$F_{T,\ air} = W = \rho_{concrete}\ g\ V = (2300\ kg/m^3)*(9.81\ m/s^2)*(0.48\ m^3) = 10{,}830\ kg.m/s^2$

$= 10{,}830\ N = 10.8\ kN$

$F_{T,\ water} = W - F_B$

$F_B = \rho_f\ gV = (1025\ kg/m^3)*(9.81\ m/s^2)*(0.48\ m^3) = 4{,}827 = 4.8\ kN$

$F_{T,\ water} = 10.8 – 4.8 = 6\ kN$

Stability of a Submerged Body

The stability of a submerged body is detected according to the positions of both the center of gravity (G) and the center of buoyancy (B), as shown in Figure 20.

An immersed neutrally buoyant body is:

(*a*) stable if the center of gravity *G* is directly below the center of buoyancy *B* of the body,

(*b*) neutrally stable if *G* and *B* are coincident, and

(*c*) unstable if *G* is directly above *B*.

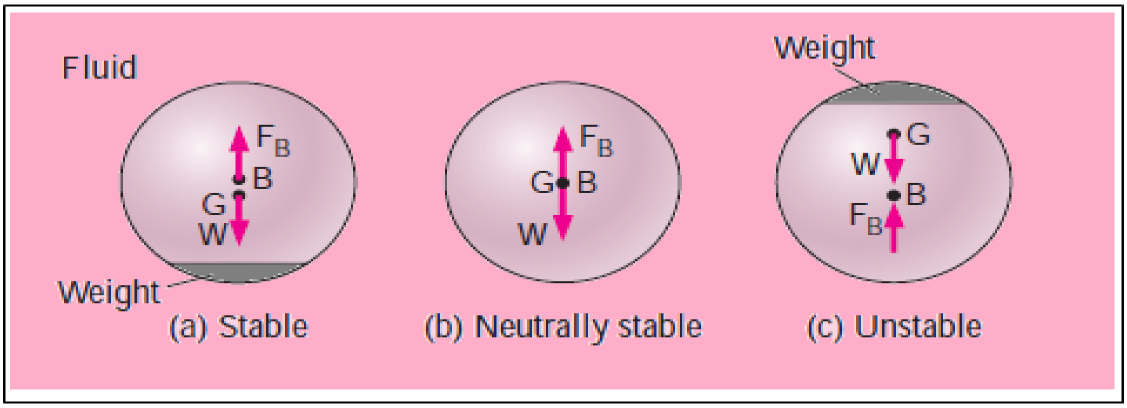


Figure 20. Stability of a Submerged Body

When the center of gravity *G* of an immersed neutrally body is not vertically aligned with the center of buoyancy *B* of the body, it is not in an equilibrium state and would rotate to its stable state, even without any disturbance, as shown in Figure 21.

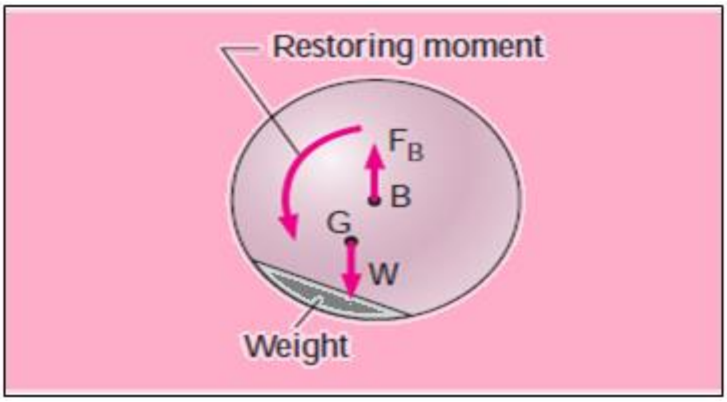


Figure 21. G is not Vertically Aligned with B

When a ship tilts, the center of buoyancy of the ship moves laterally. It may also move up or down concerning the water line.

The point at which a vertical line through the tilted center of buoyancy crosses the original center line is the metacenter M (vertical center of buoyancy).

Figure 22 shows the ship stability diagram including the center of gravity (G), the center of buoyancy (B), and the metacenter (M) with the ship upright and tilted over to one side.

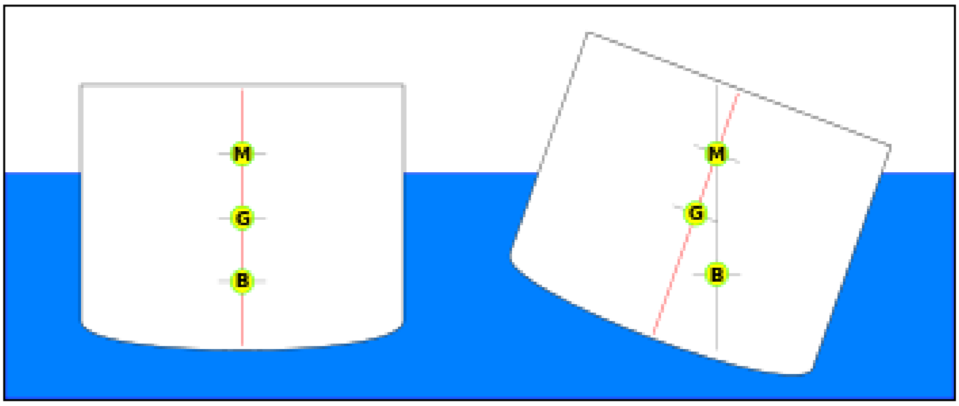


Figure 22. Ship Stability Diagram

The metacenter M is considered to be fixed for small angles of tilt (usually 0-15 degrees). At larger angles of tilt, the metacenter cannot be considered fixed, and its actual location must be found to find the ship's stability.

A floating body is stable if the body is bottom-heavy and thus the center of gravity G is below the centroid B of the body, or if point G is below the metacenter M.

However, the body is unstable if point G is above point M.

G is below B: Stable condition always.

The forces of thrust and weight are equal and inline, as shown in Figure 23 (a).

G is above B: For a small disturbance, {*taking moment about M*},

- G is below M is a stable condition, as shown in Figure 23 (b).
- G is above M is the unstable condition, as shown in Figure 23 (c).

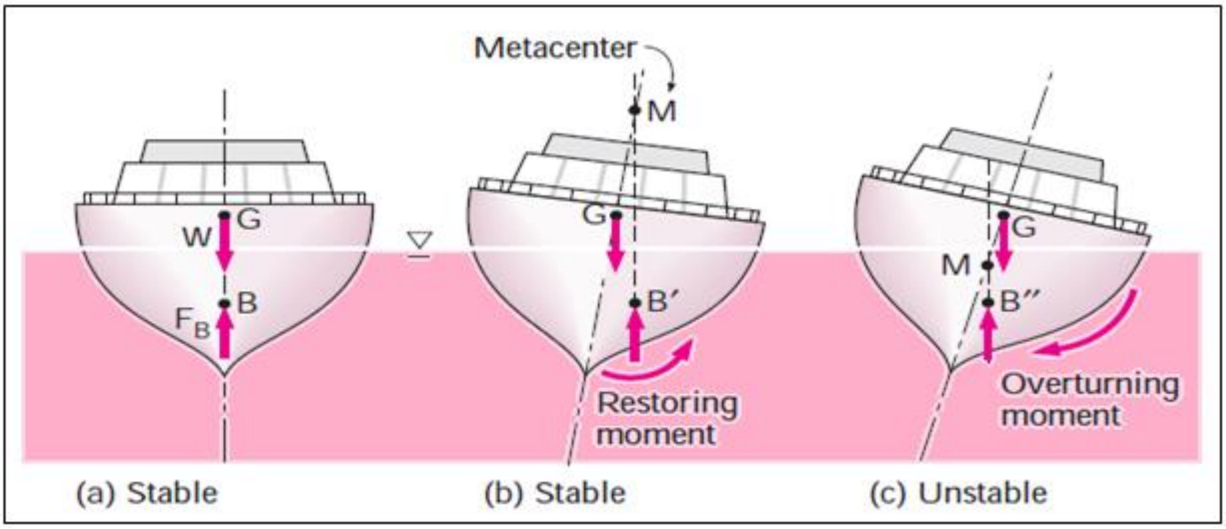


Figure 23. Stability according to G, M, and B

Figure 24 includes a summary of stability conditions.

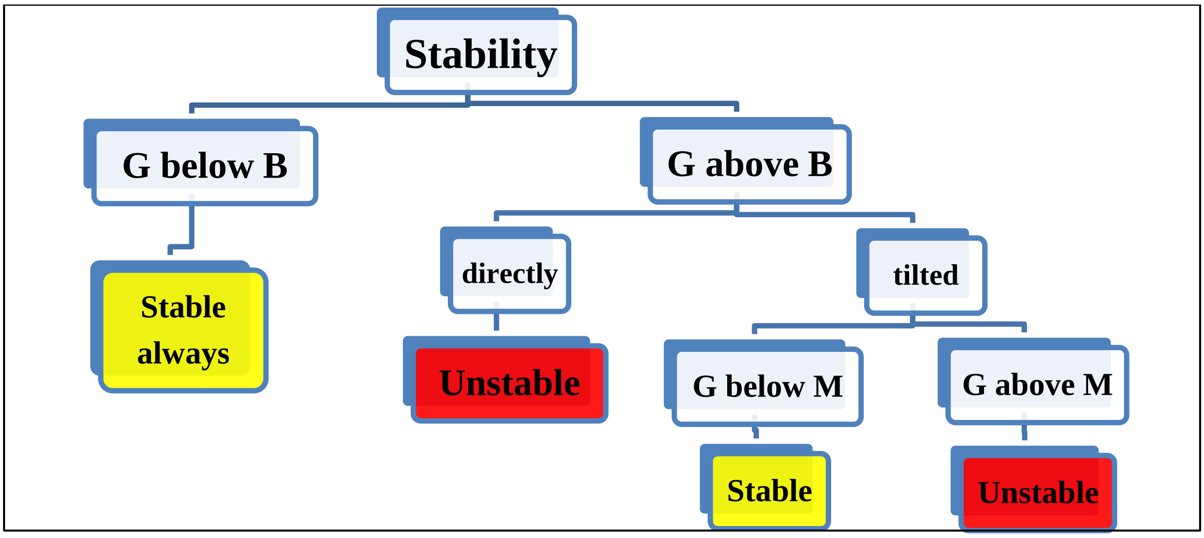


Figure 24. Stability Conditions

The metacentric height (GM) is the distance between the center of gravity of a ship and its metacenter. It is a measure of the initial static stability of a floating body.

- A larger metacentric height implies greater initial stability against overturning.
- The metacentric height also influences the natural period of rolling of a hull. For very large metacentric heights associated with short periods of roll, it will be uncomfortable for passengers.

<u>Wavelength and Amplitude</u>

<u>Wavelength:</u> The distance between two consecutive waves.

<u>Amplitude:</u> The height of the waves.

As shown in Figure 25, there are two types of problems: ends of the ship at the high (a) and low (b) of the wave resulting in flexure (bending).

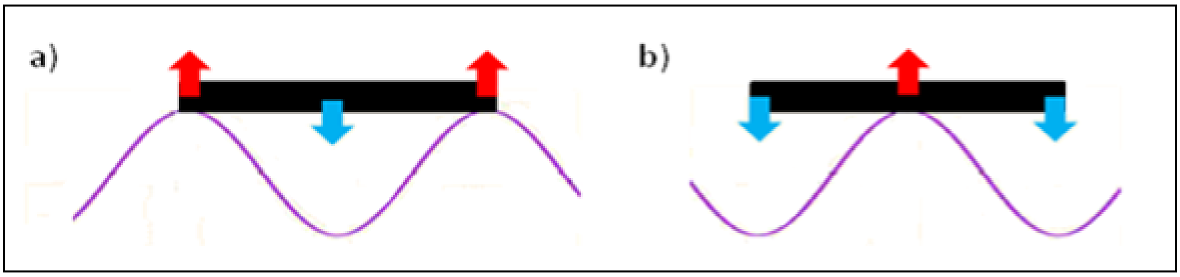


Figure 25. Problems Facing Ships

Beam-like ships are relatively long and narrow. The ship "MOL Comfort" was 316 m x 45.5 m, as shown in Figure 26.

Figure 26. Damage of the Beam-like Ship "MOL Comfort"

https://www.seanews.com.tr/why-and-how-did-the-mol-comfort-break-in-half/104726/
https://www.teccontainer.com/blog/alarming-rise-in-containers-lost-at-sea/

Example 13

A block of wood 4 m long, 2 m wide, and 1 m deep is floating in the water. The specific weight of wood is 700 kg/m^3.

1- Calculate the volume of displaced water.

2- Determine the position of the center of buoyancy.

Solution

1- Volume of block $V = 4 * 2 * 1 = 8\ m^3$

Weight of block $W = \gamma * V = 700 * 8 = 5{,}600$ kg

Floating block, $W = F_B$ $m\ g = \gamma_W\ V_d$

$\gamma\ V_t = \gamma_W\ V_d$

$V_d = \gamma\ V_t / \gamma_W$

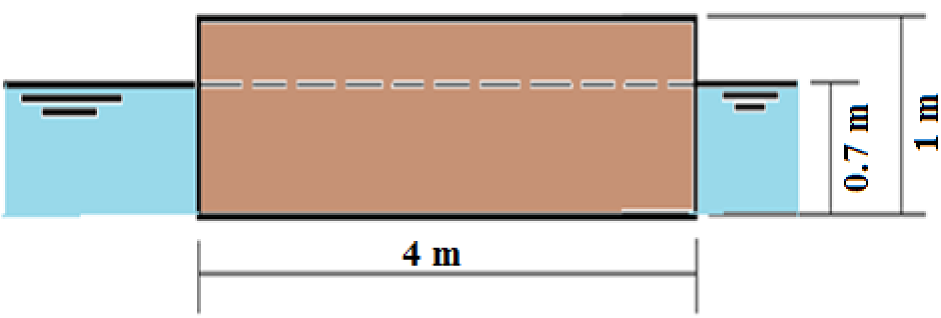
$\therefore$ Volume of displaced water $V_d = 700*8 / 1{,}000 = 5.6\ m^3$

2- Volume of the submerged section of block = $V = 5.6\ m^3$

= Submerged area * Submerged depth

Submerged depth = 5.6 / (4 * 2) = 0.7 m

$\therefore$ Centre of buoyancy B = 0.7 / 2 = 0.35 m from the base



Example 14

King Hero ordered a new crown to be made from pure gold (density = 19,200 kg/m^3). He asked Archimedes to check the crown. Archimedes found that the crown weighs 20.91 N when submerged in water, and the displaced water is $3.1*10^{-4}$ m^3.

Is the crown made from pure gold?

Solution

$\Sigma F = W - F_B$

$20.9 = \rho_c \, g \, V - \rho_w \, g \, V = (\rho_c - \rho_w) \, g \, V$

$(\rho_c - \rho_w) = 20.9 / g \, V$

$\rho_c = \rho_w + 20.9 / (9.81 * 3.1 * 10^{-4}) = 7{,}876 \text{ kg/m}^3 < 19{,}200$

∴ The crown is not made from pure gold.

Example 15

The ship in the figure weighs 300 million pounds and has dimensions of 800 ft long, 100 ft wide, and 150 ft high. $\gamma W = 62.4$ lb/ft^3.

1) Will the ship float on water?
2) How much of the boat will be submerged?

https://bankline.wordpress.com/other-classic-ships/

Solution

1) Assume fully submerged

$F_B = Vol * \gamma_W$

$F_B = (800 * 100 * 150) * 62.4 = 748.8*10^6$ lb

$F_B > W$

$\therefore$ The ship will float.

2) $W = F_B$

$300*10^6 = (800 * 100 * h) * 62.4$

$\therefore$ h = Submerged depth = 60.1 feet

Chapter 3

KINEMATICS OF FLUID FLOW

1. Types of Flow	*2. The Rate of Discharge (Q)*	*3. Continuity Equation*

The Kinematics of fluid flow studies the motion of the fluid without concerning the forces that cause this motion. That is to study the velocity and acceleration of fluid particles neglecting forces and energy considerations.

3-1 Types of Flow

Uniform Flow	**Non-Uniform Flow**
According to the effect of the distance on the flow parameters (such as velocity), the flow is uniform or non-uniform.	
When the flow parameters do not change with distance, the flow is uniform. $\frac{dv}{dx} = 0$	When the flow parameters change with distance, the flow is non-uniform. $\frac{dv}{dx} \neq 0$

Steady Flow	**Unsteady Flow**
According to the effect of time on the flow parameters (such as velocity), the flow is steady or unsteady.	
When the flow parameters do not change with time, the flow is steady. $\frac{dv}{dt} = 0$	When the flow parameters change with time, the flow is unsteady. $\frac{dv}{dt} \neq 0$

Laminar (Streamline) Flow	**Turbulent Flow**
According to the type of motion of liquid particles, the flow is laminar (streamline) or turbulent.	
When the paths of liquid particles do not cross each other, the flow is laminar.	When the paths of liquid particles cross each other, the flow is turbulent.

Rotational (Vortex) Flow	**Irrotational Flow**
According to the rotation of liquid particles about axes during their motion, the flow is rotational (vortex) or irrotational.	
When the liquid particles rotate about axes during their motion, the flow is rotational (vortex).	When the liquid particles do not rotate about axes during their motion, the flow is irrotational.

One Dimensional Flow	**Two Dimensional Flow**	**Three Dimensional Flow**
According to the number of directions along which the flow parameters (such as velocity) change, the flow is one, or two, or three-dimensional.		
When the flow parameters change in 1 direction, the flow is a 1-dimensional flow	When the flow parameters change in 2 directions, the flow is a 2-dimensional flow	When the flow parameters change in 3 directions, the flow is a 3-dimensional flow

3-2 The Rate of Discharge (Q)

It is the quantity of liquid flowing through a section of the conduit per unit of time.

$$Q = V / t = (A\ l) / t = A * (l / t)$$

$$\mathbf{Q = A\ v}$$

Where: Q: The discharge (rate of flow).

A: The cross-sectional area of the conduit.

v: The average velocity of the liquid.

3-3 Continuity Equation

Consider a liquid flows through a conduit, as shown in Figure 1, and the values of cross-sectional area, velocity, and density of liquid are A_1, v_1, ρ_1, and A_2, v_2, ρ_2 for the two sections respectively.

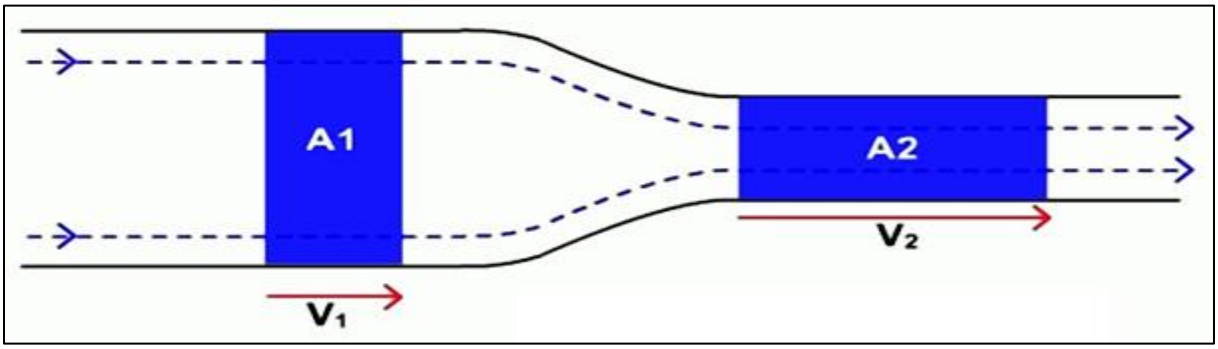


Figure 1. Continuity Equation

From the principle of conservation of mass, mass of liquid flowing per unit time through section 1 = mass of liquid flowing per unit time through section 2 + change of mass of liquid per unit time between sections 1 & 2.

For a steady flow, there is no change in liquid parameters with time. There is no change in the mass of liquid with time. Thus, the mass of liquid flowing per unit time through section 1 = mass of liquid flowing per unit time through section 2.

$$M_1 / t = M_2 / t$$

But, $M / t = \rho V / t = (\rho A l) / t = \rho * A * (l / t) = \rho A v$

Then, $\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

Because liquids are incompressible fluids, the density ρ is constant.

$$\mathbf{A_1 v_1 = A_2 v_2 = A_3 v_3 = \ldots\ldots\ldots\ldots = Constant}$$

$$\mathbf{Q_1 = Q_2 = Q_3 = \ldots\ldots\ldots\ldots = Constant}$$

Example 1

Water flows through a pipe of 10 cm diameter with a velocity of 10 m/sec.

1- Determine the discharge.

2- If the diameter of the pipe is 20 cm, determine the velocity.

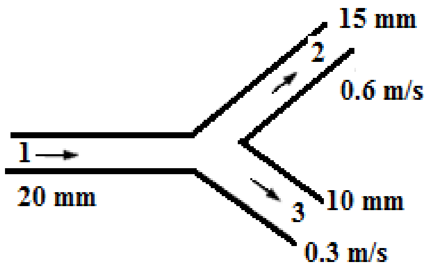
Solution

1- $Q = A\,v = (\pi\,(0.1)^2 / 4) * 10 = 0.078\ m^3/sec$

2- $v = Q / A = 0.078 / (\pi\ (0.2)^2 / 4) = 2.48\ m/sec$

Example 2

As shown in the figure, water flows through pipe (1) that branches into two pipes (2) and (3).



1) Determine the rate of flow in cm^3/s for the pipe (1).
2) Find the velocity in m/s for the pipe (1).

Solution

1) $Q_1 = Q_2 + Q_3 = (A_2\ v_2) + (A_3\ v_3)$

$Q_1 = \{(\pi\,(1.5)^2 / 4) * 60\} + \{(\pi\,(1)^2 / 4) * 30\}$

$= 129.6\ cm^3/sec$

2) $v_1 = Q_1 / A_1 = 129.6 / (\pi\ (2)^2 / 4)$

$v_1 = 41.25\ cm/sec = 0.41\ m/sec$

Chapter 4

DYNAMICS OF FLUID FLOW

For the dynamics of fluid flow, or hydrodynamics, the fluid motion is studied including the force and energy considerations.

4-1 Types of Energy

I- Potential Energy

It is the energy a fluid particle possesses due to its position for an arbitrary datum.

II- Pressure Energy

It is the energy possessed by a fluid particle due to its pressure.

III- Kinetic Energy

It is the energy a fluid particle possesses due to its motion or velocity.

4-2 Euler's Equation

Assumptions and Limitations

1- The fluid is ideal (non-viscous or no friction losses).

2- The fluid is incompressible (ρ is constant).

3- The flow is steady.

4- The velocity of flow is uniform over the section.

5- Only the gravity and pressure forces are considered.

<u>The Equation</u>

For a steady flow of an ideal fluid, consider an element AB of the fluid, as shown in Figure 1.

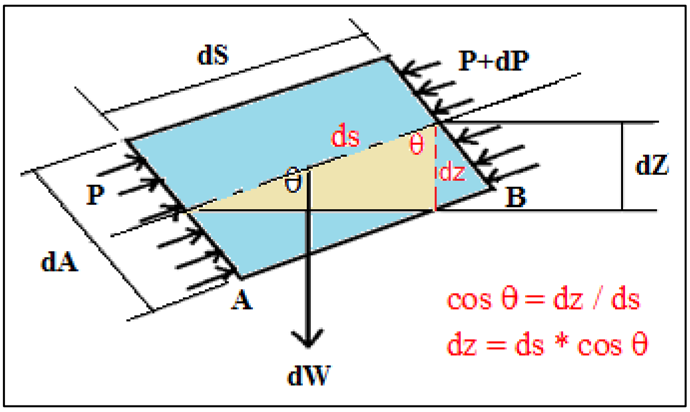


Figure 1. Steady Flow of an Ideal Fluid

<u>Where:</u> ds, dA: length and cross-sectional area of the fluid element.

dW : Weight of the fluid element.

p : Pressure of the fluid element at A.

p + dp: Pressure of the fluid element at B.

<u>Applying Newton's second law of motion in the direction of flow:</u>

$$\sum F = M * a$$

$$\sum F = P\ dA - (P + dP)\ dA - dW \cos\theta$$

$$dW = M\ g = \rho\ V\ g = \rho\ dA\ ds\ g$$

<u>Thus,</u> $\sum F = -\ dP\ dA - \rho\ dA\ ds\ g \cos\theta$ .... (1)

$$M * a = (\rho V) * a = \rho\, dA\, ds * a$$

And $a = \frac{dv}{dt} = \frac{dv}{ds} * \frac{ds}{dt} = \frac{dv}{ds}\, v$

$$a * ds = v * dv$$

Then, $M * a = \rho\, dA\, v\, dv$ .... (2)

(1) = (2), $- dP\, dA - \rho\, dA\, ds\, g \cos\theta = \rho\, dA\, v\, dv$

÷ dA, $- dP - \rho\, g\, ds \cos\theta = \rho\, v\, dv$

But, $ds \cos\theta = dz$

Thus, $- dP - \rho\, g\, dz = \rho\, v\, dv$

$$\rho\, g\, dz + dP + \rho\, v\, dv = 0$$

÷ ρ $g\, dz + dP/\rho + v\, dv = 0$

÷ g $dz + dP/\rho g + v\, dv/g = 0$

$v\, dv = ½\, d(v^2)$ $dz + dP/\rho g + dv^2/2g = 0$

4-3 Bernoulli's Equation

Integrating Euler's equation, we get Bernoulli's equation.

$$\mathbf{Z + (P/\rho g) + (v^2/2g) = Constant}$$

OR

$$\mathbf{Z + (P/\gamma) + (v^2/2g) = Constant}$$

Where: Z: Potential energy per unit weight of the fluid (or potential head for an arbitrary datum).

P / γ: Pressure energy per unit weight of fluid, or pressure head.

$v^2/2g$: Kinetic energy per unit weight of fluid, or velocity head.

Applying Bernoulli's equation between two points along the flow of the fluid, *we get:*

$$Z_1 + (P_1/\gamma) + (v_1^2/2g) = Z_2 + (P_2/\gamma) + (v_2^2/2g) = \text{Constant}$$

$* \rho g$ $$\rho g Z + P + \rho v^2 / 2 = \text{Constant}$$

The variation of pressure with elevation in steady, incompressible flow along a straight line is the same as that in the stationary fluid, as shown in Figure 2.

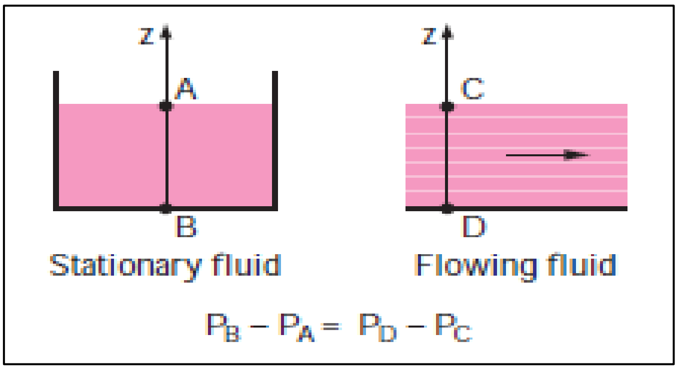


Figure 2. Variation of Pressure with Elevation

The kinetic and potential energies of the fluid can be converted to pressure energy (and vice versa) during flow, causing the pressure to change, as shown in Figure 3.

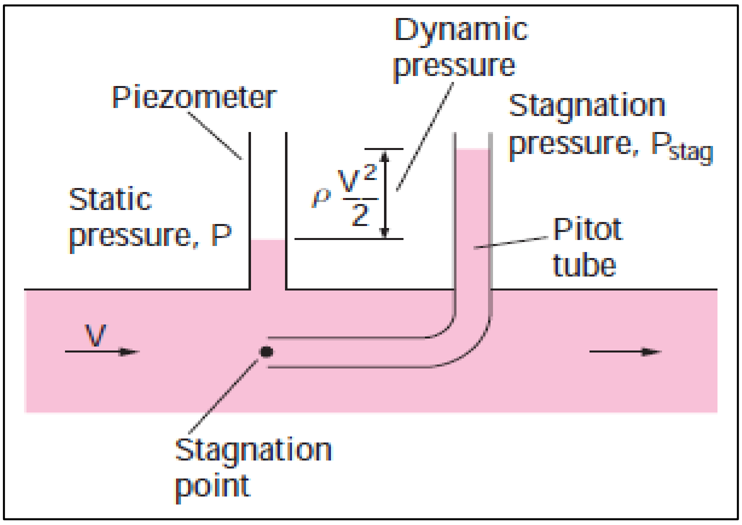


Figure 3. Conversion of Energy

- P is the static pressure (it does not incorporate any dynamic effects).
- $\rho V^2/2$ is the dynamic pressure; it represents the pressure rise when the fluid in motion is brought to a stop symmetrically.
- $\rho\ g\ z$ is the hydrostatic pressure (which is not pressure in a real sense since its value depends on the reference level selected); it accounts for the elevation effects, i.e., of fluid weight on pressure.
- The sum of the static, dynamic, and hydrostatic pressures is called the total pressure. (Bernoulli equation states that the total pressure along a streamline is constant).
- The sum of the static and dynamic pressures is called the stagnation pressure, and it is expressed as $\mathbf{P_{stag} = P + \rho\ v^2 / 2}$
- The stagnation pressure represents the pressure at a point where the fluid is brought to a complete stop symmetrically.

4-4 Total Energy Line (TEL) and Hydraulic Grade Line (HGL)

Total Energy = Potential Head + Pressure Head + Velocity Head

Piezometric Head = Potential Head + Pressure Head

For different sections along the fluid flow, an arbitrary datum is chosen, as shown in Figure 4.

The potential, pressure, and velocity heads are assigned on a vertical line through each section above the datum using an adequate scale.

The line between points representing the total head is the total energy line (TEL or EGL).

The line between points representing the piezometric head is the hydraulic grade line (HGL).

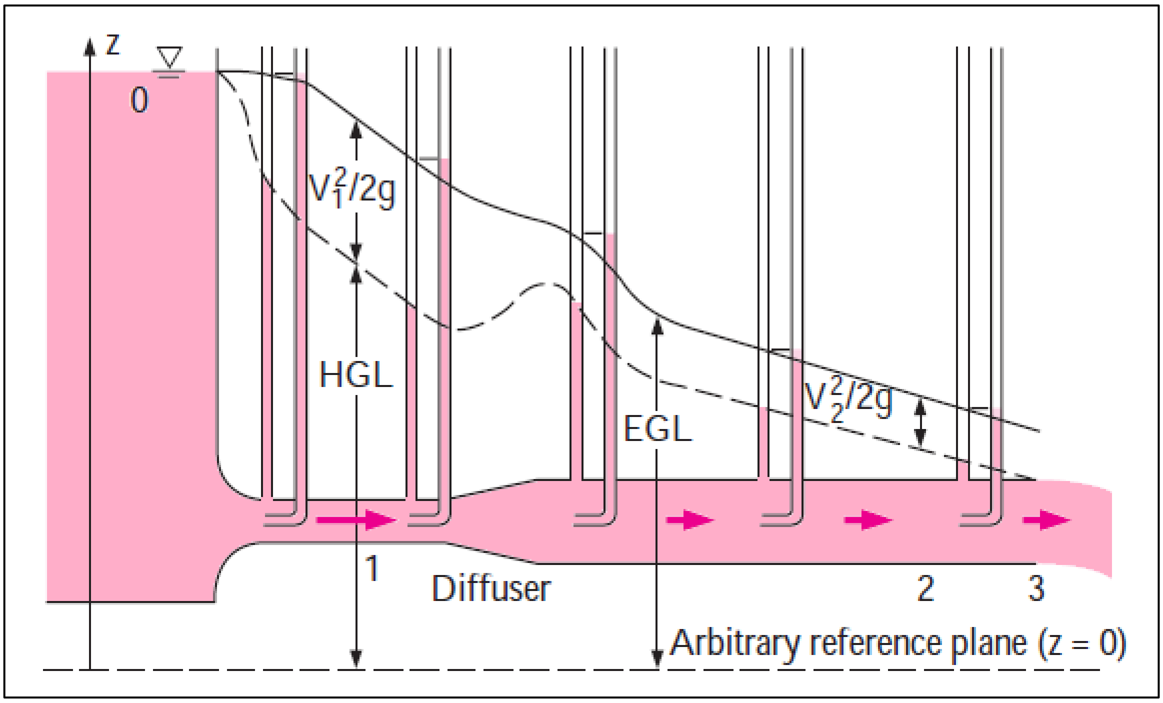


Figure 4. TEL (EGL) and HGL

Notes

- For stationary bodies such as reservoirs or lakes, the EGL and HGL coincide with the free surface of the liquid. The elevation of the free surface z in such cases represents both the EGL and the HGL since the velocity is zero and the static pressure (gage) is zero.
- For open-channel flow, the HGL coincides with the free surface of the liquid, and the EGL is a distance $v^2/2g$ above the free surface.
- The EGL is always a distance $v^2/2g$ above the HGL.
- At a pipe exit, the pressure head is zero (atmospheric pressure); thus, the HGL coincides with the pipe outlet (3 in Fig. 4).
- The energy loss due to frictional effects causes the EGL and HGL to slope downward in the direction of flow. The slope is a measure of the head loss in the pipe. A component that generates significant frictional effects such as a valve causes a sudden drop in both EGL and HGL at that location.

$$Z_1 + (P/\gamma)_1 + (v^2/2g)_1 = Z_2 + (P/\gamma)_2 + (v^2/2g)_2 + h_L$$

- The fluid flows from the point of high total energy to that of low total energy.
- In an ideal flow, EGL is horizontal, and its height remains constant. This is also the case for HGL when the flow velocity is constant, as shown in Figure 5.

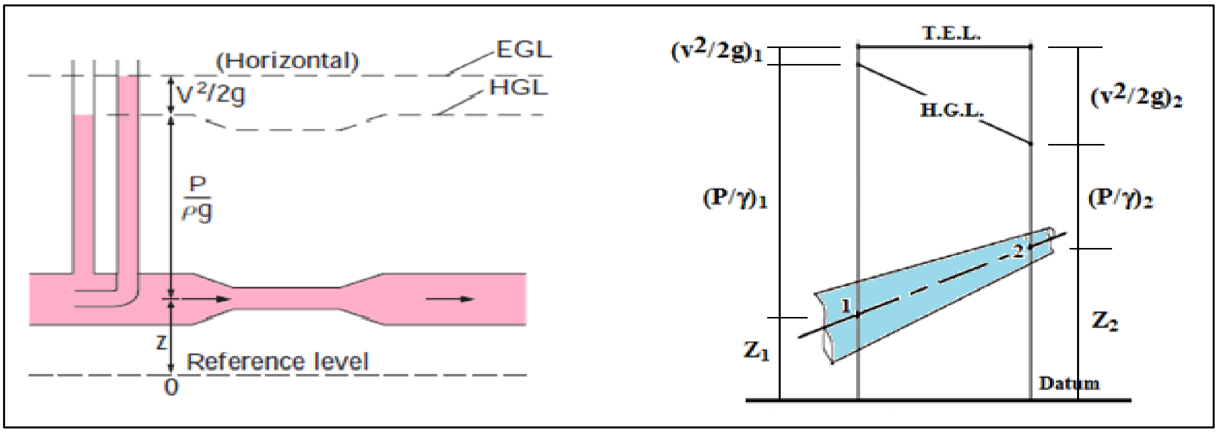


Figure 5. Horizontal TEL and HGL

- A steep jump occurs in EGL and HGL whenever mechanical energy is added to the fluid (by a pump, for example). Likewise, a steep drop occurs in EGL and HGL whenever mechanical energy is removed from the fluid (by a turbine, for example), as shown in Figure 6.
- The pressure (gage) of fluid is zero at locations where the HGL intersects the fluid. The pressure in a flow section that lies above the HGL is negative (vacuum), and the pressure in a section that lies below the HGL is positive, as shown in Figure 7.

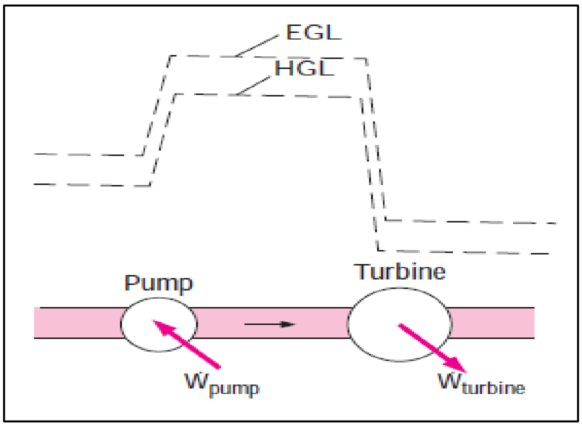


Figure 6. Jump and Drop-in EGL & HGL

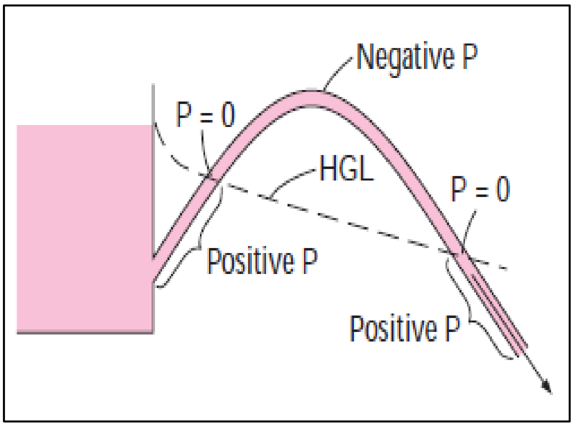
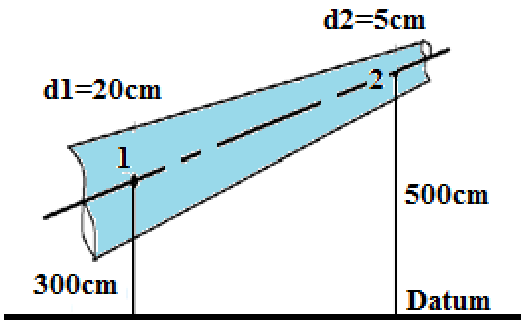


Figure 7. Zero, -ve, and +ve Pressure

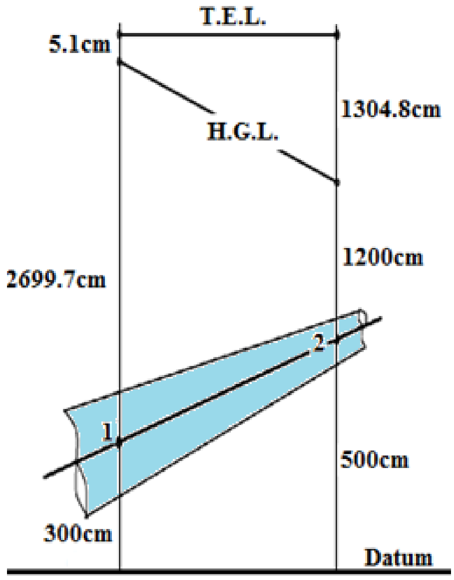
Example 1



As shown in the figure, the diameter of a pipe changes from 20 cm at a section 3 m above a datum to 5 cm at another section 5 m above the datum. In the second section, the pressure of water is 1.2 kg/cm^2, and the velocity of the flow is 16 m/s.

Determine the pressure in the first section.

Solution



$A_1 = \pi\, d^2 / 4 = \pi\,(20)^2 / 4 = 314.16\ cm^2$

$Z_1 = 3\ m = 300\ cm$

$A_2 = \pi\, d^2 / 4 = \pi\,(5)^2 / 4 = 19.63\ cm^2$

$Z_2 = 5\ m = 500\ cm$

$P_2 = 1{,}200\ g/cm^2$

$V_2 = 16\ m/s = 1{,}600\ cm/s$

$A_1\, v_1 = A_2\, v_2$

$V_1 = (19.63 * 1{,}600)/314.16 \sim 100\ cm/s$

$\gamma_w = 1\ gm/cm^3$

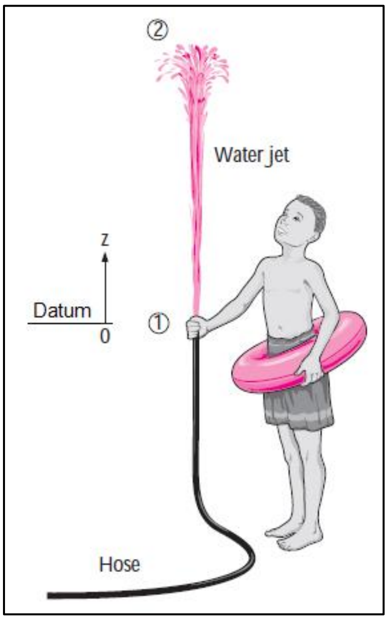
Bernoulli's equation:

$Z_1 + (P/\gamma)_1 + (v^2/2g)_1 = Z_2 + (P/\gamma)_2 + (v^2/2g)_2$

$300 + (P_1/1) + [(100)^2 / (2 * 981)] = 500 + (1200/1) + [(1600)^2 / (2 * 981)]$

$P_1 = 2699.7\ gm/cm^2$

Example 2



Water is flowing from a hose attached to a water main at 400 kPa gage, as shown in the figure. A child places his thumb to cover most of the hose outlet, causing a thin jet of high-speed water to emerge.

What is the maximum height that the jet could achieve if the hose is held upward?

Solution

$Z_1 + (P/\gamma)_1 + (v^2/2g)_1 = Z_2 + (P/\gamma)_2 + (v^2/2g)_2$

$Z_1 = 0$ $v_1 = 0$ $P_2 = 0$ $v_2 = 0$

$(P/\gamma)_1 = Z_2$

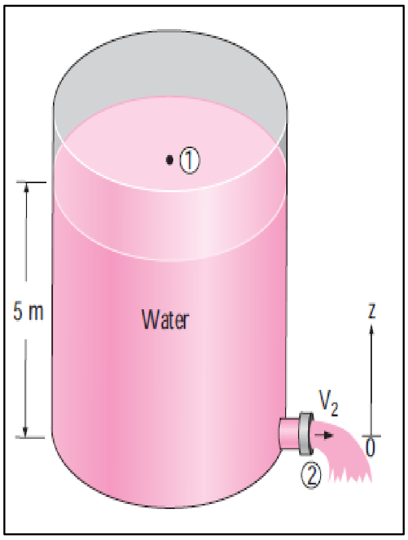
$Z_2 = (400*1,000) / 9810 = 40.8$ m

Example 3

A large tank open to the atmosphere is filled with water to a height of 5 m from the outlet tap, as shown in the figure. A tap near the bottom of the tank is now opened, and water flows out from the smooth and rounded outlet.

Determine the water velocity at the outlet.

Solution



$Z_1 + (P/\gamma)_1 + (v^2/2g)_1 = Z_2 + (P/\gamma)_2 + (v^2/2g)_2$

$Z_1 = 5$ m

$v_1 = 0$ & $P_1 = 0$

$Z_2 = 0$ & $P_2 = 0$

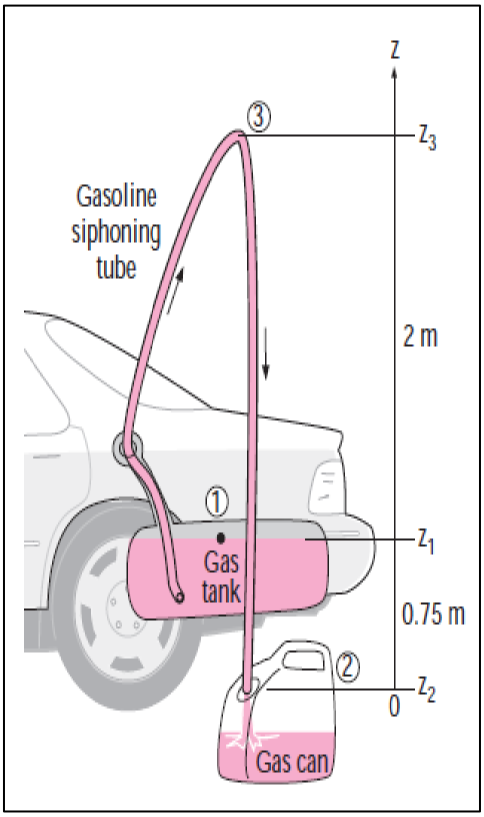
$5 = (v^2/2g)_2$

$v_2^2 = 10$ g

$v_2 = \sqrt{10 * 9.81} = 9.9$ m/s

Example 4

It is required to siphon gasoline out of a car (the density of gasoline is 750 kg/m^3), as shown in the figure. The siphon is a tube (hose) of 5 mm in diameter. The difference in pressure between point 1 (at the free surface of the gasoline in the tank) and point 2 (at the outlet of the tube) causes the liquid to flow from the higher to the lower elevation. Point 2 is located 0.75 m below point 1 and point 3 is located 2 m above point 1. The frictional losses in the siphon are to be disregarded.



1) Determine the minimum time to withdraw 4 L of gasoline from the tank to the can.
2) Find the pressure at point 3.

Solution

To start the siphon, it is necessary to insert one siphon end in the full gas tank, fill the hose with gasoline via suction, and then place the other end in a gas can below the level of the gas tank.

1) $Z_1 + (P/\gamma)_1 + (v^2/2g)_1 = Z_2 + (P/\gamma)_2 + (v^2/2g)_2$

$Z_1 = 0.75$ m $\quad v_1 = 0 \quad P_1 = 0$

$Z_2 = 0 \quad P_2 = 0$

$0.75 = (v^2/2g)_2$

$v_2^2 = 1.5$ g $\quad v_2 = \sqrt{1.5 * 9.81} = 3.84$ m/s

$Q_2 = A_2 * v_2 = \{\pi (0.005)^2/4\} * 3.84 = 7.53*10^{-5}$ m^3/s = 0.0753 L/s

t = V / Q = 4 / = 0.0753 = 53.1 s

2) $Z_2 + (P/\gamma)_2 + (v^2/2g)_2 = Z_3 + (P/\gamma)_3 + (v^2/2g)_3$

$Z_2 = 0 \quad P_2 = 0 \quad v_2 = v_3 \; Z_3 = 2.75$ m

$0 = 2.75 + (P/\gamma)_3$

$(P/\gamma)_3 = -2.75 \quad P_3 = -2.75 * 750 * 9.81 = -20{,}233.13$ N/m^2 (Pa)

4-5 Applications of Bernoulli's Equation

Three applications will be considered for flow measurements in pipes: venturi meter, pitot tube, and orifice meter.

I- Venturimeter

It is a device for measuring the discharge of a liquid flowing in a pipe. It consists of three parts: convergent cone or inlet, throat, and divergent cone or outlet, as shown in Figure 8.

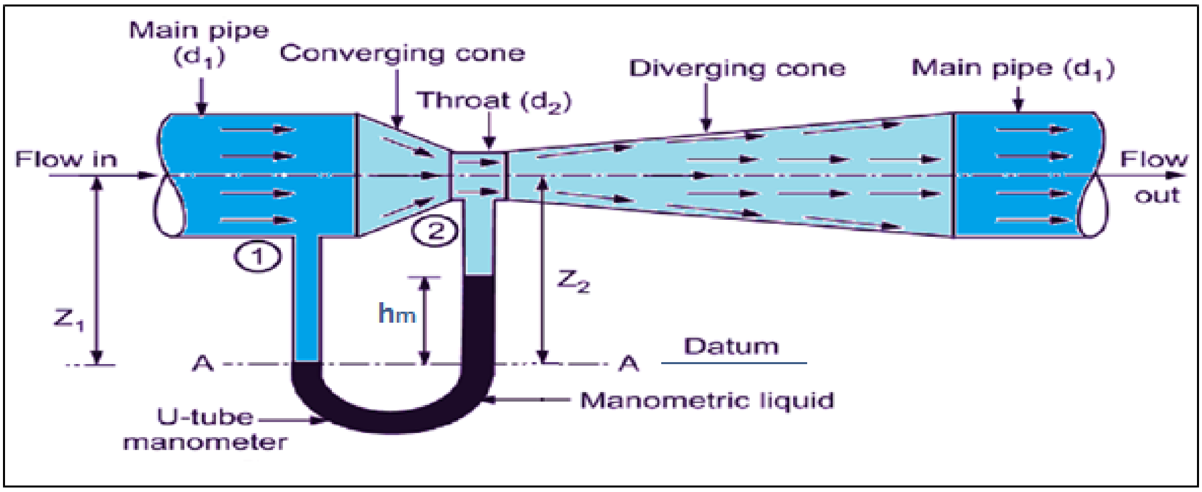


Figure 8. Venturimeter

https://electricalworkbook.com/venturimeter/

- The inlet is a short pipe that converges from the pipe diameter d_1 to a smaller diameter d_2. This convergent pipe converts the pressure head into a velocity head.
- The throat is a small circular pipe with a constant diameter of d_2.
- The outlet is a longer pipe that diverges from the throat diameter d_2 to the pipe diameter d_1. This divergent pipe converts the velocity head into a pressure head.

For ideal fluid, applying Bernoulli's equation between sections (1) and (2) representing the inlet and throat respectively,

$$Z_1 + (P_1 / \gamma) + (v_1^2 /2g) = Z_2 + (P_2 / \gamma) + (v_2^2 /2g)$$

$[Z_1 + (P_1 / \gamma)] - [Z_2 + (P_2 / \gamma)] = (v_2^2 - v_1^2) / 2g$

$H = (v_2^2 - v_1^2) / 2g$

<u>Where:</u> $[Z_1 + (P_1 / \gamma)] - [Z_2 + (P_2 / \gamma)] = H$ = change in piezometric head.

A manometer can be used to measure the change in piezometric head H.

$Z_1 + (P_1 / \gamma) = (Z_2 - h_m) + (P_2 / \gamma) + (\gamma_m h_m / \gamma)$

$Z_1 + (P_1 / \gamma) - Z_2 - (P_2 / \gamma) = (\gamma_m h_m / \gamma) - h_m$

$[Z_1 + (P_1 / \gamma)] - [Z_2 + (P_2 / \gamma)] = h_m [(\gamma_m / \gamma) - 1]$

$H = h_m [(\gamma_m / \gamma) - 1]$

$H = h_m [(\gamma_m / \gamma) - 1] = (v_2^2 - v_1^2) / 2g$

$(v_2^2 - v_1^2) = 2\, g\, H$

$Q = A_1 v_1 = A_2 v_2$ $\qquad$ $v_1 = A_2 v_2 / A_1$

$$v_2^2 - \frac{A_2^2\, v_2^2}{A_1^2} = 2\, g\, H$$

$$v_2^2\, (1 - \frac{A_2^2}{A_1^2}) = 2\, g\, H$$

$$v_2^2\, (\frac{A_1^2 - A_2^2}{A_1^2}) = 2\, g\, H$$

$$v_2^2 = \left(\frac{A_1^2}{A_1^2 - A_2^2}\right) * 2\, g\, H$$

$$v_2 = \frac{A_1 \sqrt{2\, g\, H}}{\sqrt{A_1^2 - A_2^2}}$$

$Q = A_2 v_2$

$$Q = \frac{A_1\, A_2 \sqrt{2\, g\, H}}{\sqrt{A_1^2 - A_2^2}}$$

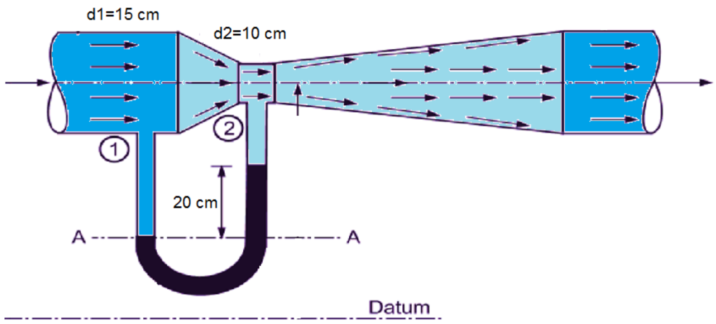
This is the equation of the venturimeter for measuring the discharge of ideal fluid flowing in a pipe.

Example 5

A venturi meter of 15 cm inlet diameter and 10 cm throat is laid horizontally in a pipe to measure the flow of oil of 0.9 specific gravity. The reading of a mercury manometer is 20 cm.

Calculate the discharge in L/min.

Solution



For inlet,

$A_1 = (\pi\, d_1^2)/4 = (\pi * 15^2)/4 = 176.7\ cm^2$

For throat,

$A_2 = (\pi\, d_2^2)/4 = (\pi * 10^2)/4 = 78.54\ cm^2$

$H = h_m\,[(\gamma_m / \gamma) - 1] = 20\,[(13.6 / 0.9) - 1] = 282.2$ cm of oil

$$Q = \frac{A_1 A_2 \sqrt{2gH}}{\sqrt{(A_1^{\,2} - A_2^{\,2})}}$$

$$Q = \frac{(176.7 x 78.54)\sqrt{2 x 981 x 282.2}}{\sqrt{(176.7)^2 - (78.54)^2}}$$

$Q = 65238.2\ cm^3/s$

(* 60/1000) $Q = 3914.3$ L/min

Example 6

A 30 cm x 15 cm venturimeter is provided to a vertical pipeline carrying oil with 0.9 specific gravity. The flow direction is upwards. The difference in elevation between the inlet and throat is 30 cm. The reading of a mercury manometer is 25 cm.

1- Calculate the discharge.

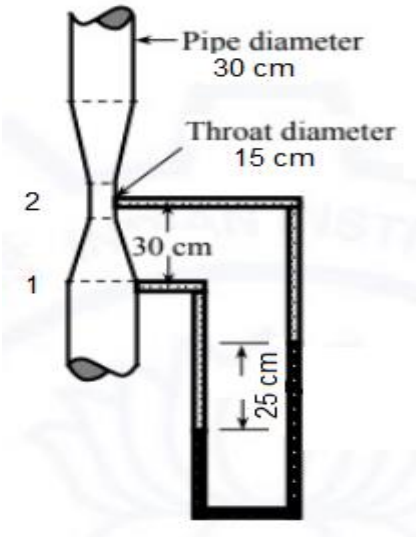
2- Determine the pressure head between the inlet and throat.

Solution

(1) For inlet, $A_1 = (\pi\, d_1^2)/4 = (\pi * 30^2)/4 = 706.86\ cm^2$

For throat, $A_2 = (\pi\, d_2^2)/4 = (\pi * 15^2)/4 = 176.71\ cm^2$

$H = h_m\, [(\gamma_m / \gamma) - 1] = 25\, [(13.6 / 0.9) - 1] = 352.8$ cm of oil



$$Q = \frac{A_1 A_2 \sqrt{2gH}}{\sqrt{(A_1^2 - A_2^2)}}$$

$$Q = \frac{(706.86 x 176.71)\sqrt{2 x 981 x 352.8}}{\sqrt{(706.86)^2 - (176.71)^2}}$$

$Q = 151840.6\ cm^3/s$

(2) $[Z_1 + (P_1 / \gamma)] - [Z_2 + (P_2 / \gamma)] = H$

$(P_1 / \gamma) - (P_2 / \gamma) + (Z_1 - Z_2) = 352.8$

$Z_1 - Z_2 = 0 - 30 = -30$

$(P_1 / \gamma) - (P_2 / \gamma) = 352.8 + 30 = 382.8$ cm of oil

Exercises

(1) Resolve example (6) if the flow is downwards.

(2) Resolve examples (5) and (6) by applying only Bernoulli's equation and without using the equation of the venturimeter.

Important Notes

1- Venturimeter for a real fluid flow.

In the case of a real fluid, there is energy loss between any sections decreasing the values of velocity. Thus, a coefficient of venturimeter (or the coefficient of discharge) C_d is employed.

$$\mathbf{Q_R = C_d\, Q_I}$$

$C_d = 0.95 - 0.99$ (It is designed to minimize energy losses.)

It may be noted that,

$$C_d = Q_R / Q_I = (v_R\, A) / (v_I\, A) = (v_R / v_I)$$

$$\mathbf{V_R = C_d\, v_I}$$

2- The reading of the manometer attached to a venturimeter is constant. It does not depend on the position of the venturi meter. as shown in Figure 9.

$$[Z_1 + (P_1 / \gamma)] - [Z_2 + (P_2 / \gamma)] = H = (v_2^2 - v_1^2) / 2g$$

H = Constant, as the velocities v_1 & v_2 are constant for continuous flow.

* *For horizontal venturi meter:*

The reading of manometer h_m is employed to get H, as discussed before.

H = Potential Head + Pressure Head

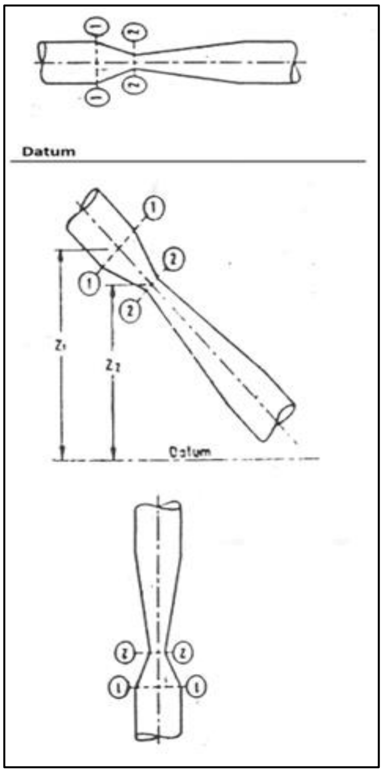


Figure 9. Constant Reading of Manometer

* *For inclined venturi meter:*

When the venturi meter is inclined, the potential head changes. Consequently, pressure head changes also in a manner such that H is still constant. If the potential head decreases, then the pressure head increases such that their sum H is still constant.

* *For vertical venturi meter:*

When the venturi meter is fixed vertically, the potential head changes according to the direction of flow (upwards or downwards). Consequently, pressure head changes such that their sum H is still constant.

3- Negative pressure at the throat of a venturi meter.

At the throat of a venturi meter, the velocity is maximum because it has a minimum cross-sectional area and consequently the pressure is minimum. Thus, the pressure may be zero or even negative. When this negative pressure reaches the value of vapor pressure of the liquid flowing in the pipe, the liquid evaporates. So, the flow becomes discontinuous due to the existing vapor. Cavitation takes place as the liquid evaporates, and the vapor condenses to a liquid, and so on.

To avoid cavitation, the pressure at the throat of a venturimeter must not reach the value of the vapor pressure of the flowing liquid.

II- Pitot Tube

It is a device for measuring the velocity of a liquid flowing in a pipe or an open channel. As shown in Figure 10, it consists of a glass tube bent at 90° with a short length. The lower short end of the pitot tube is put to face the direction of flow. The liquid rises in the tube due to the pressure of the flowing liquid. The rise of liquid is measured to calculate the velocity of the flowing liquid.

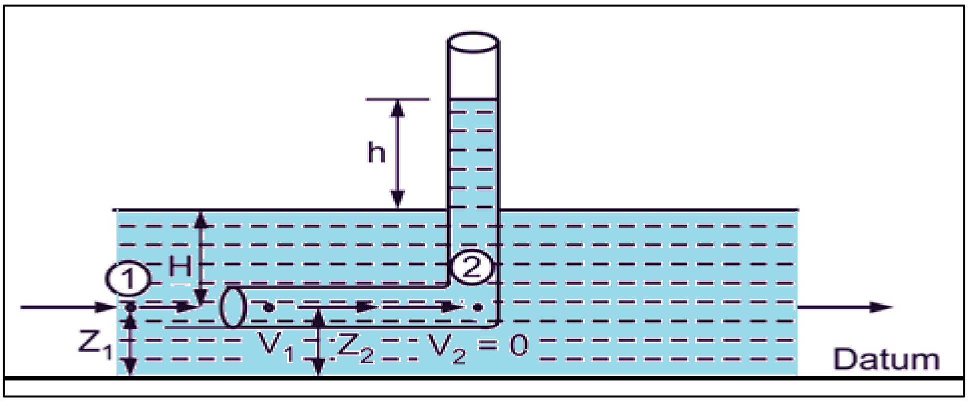


Figure 10. Pitot Tube

https://electricalworkbook.com/pitot-tube/

For ideal fluid, applying Bernoulli's equation between sections (1) and (2) as shown in the figure,

$$Z_1 + (P_1 / \gamma) + (v_1^2 / 2g) = Z_2 + (P_2 / \gamma) + (v_2^2 / 2g)$$

$Z_1 = Z_2 = 0$ At the same level.

$v_2 = 0$

$$(P_1 / \gamma) + (v_1^2 /2g) = (P_2 / \gamma)$$

$$(v_1^2 /2g) = (P_2 / \gamma) - (P_1 / \gamma)$$

$(P_1 / \gamma) = H$ & $(P_2 /\gamma) = H + h$

$(P_2 /\gamma) - (P_1 / \gamma) = H + h – H = h$

$$(v_1^2 /2g) = h$$

$$\boldsymbol{v_1 = \sqrt{2\,g\,h}}$$

Where h is the rise of flowing liquid in the pitot tube above the surface.

This is the equation of the pitot tube for measuring the velocity of ideal fluid flowing in a pipe or an open channel.

For the case of a real fluid, $\mathbf{Q_R = C_d\, Q_I}$

$C_d \sim 1$ (The velocity becomes zero rapidly upon entry to the tube with negligible energy losses.)

III- <u>Orifice Meter</u>

It is a device for measuring the discharge of a liquid flowing in a pipe. As shown in Figure 11, it consists of a plate having a sharp-edged circular orifice (hole).

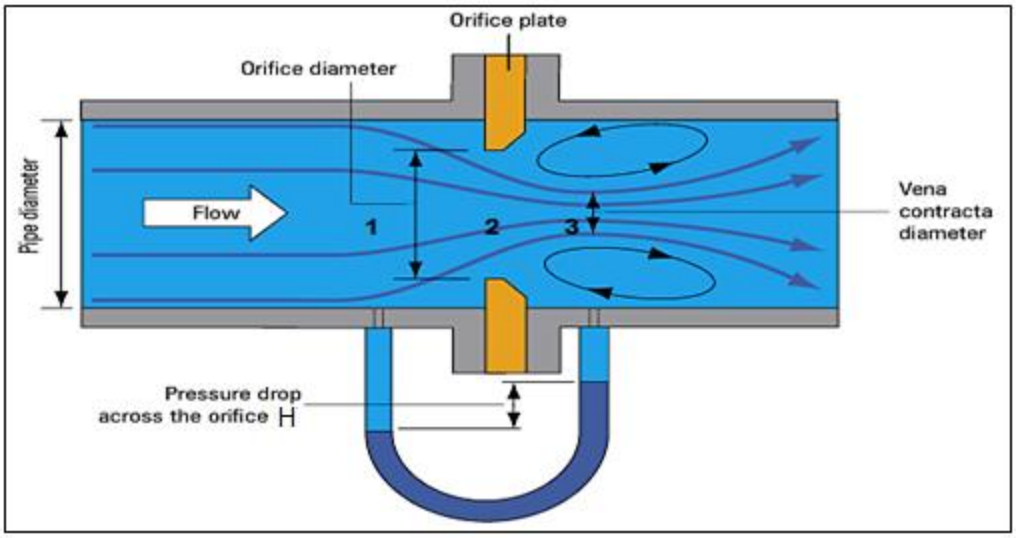


Figure 11. Orifice Meter

https://instrumentationtools.com/what-is-an-orifice-meter/

For ideal fluid, applying Bernoulli's equation between sections (1) and (2) representing the pipe and orifice respectively,

$$Z_1 + (P_1 / \gamma) + (v_1^2 /2g) = Z_2 + (P_2 / \gamma) + (v_2^2 /2g)$$

$$[Z_1 + (P_1 / \gamma)] - [Z_2 + (P_2 / \gamma)] = H = (v_2^2 - v_1^2) /2g$$

Where: H is the change in the piezometric head.

Then,

$$(v_2^2 - v_1^2) = 2 g H$$

$$(Q^2 / A_2^2) - (Q^2 /A_1^2) = 2 g H$$

$$Q^2 [(1/A_2^2) - (1/A_1^2)] = 2 g H$$

$$Q^2 = [(A_1^2 A_2^2) / (A_1^2 - A_2^2)] (2 g H)$$

$$Q = \frac{A_1 A_2 \sqrt{2 g H}}{\sqrt{A_1^2 - A_2^2}}$$

This is the equation of an orifice for measuring the discharge of ideal fluid flowing in a pipe.

In the case of a real fluid, there is energy loss between any sections decreasing the values of velocity. Thus, a coefficient of discharge C_d is employed, which accounts for both velocity and contraction.

$$\mathbf{Q_R = C_d\, Q_I}$$

C_d = 0.65: 0.67 (It is simple and cheap with high energy losses.)

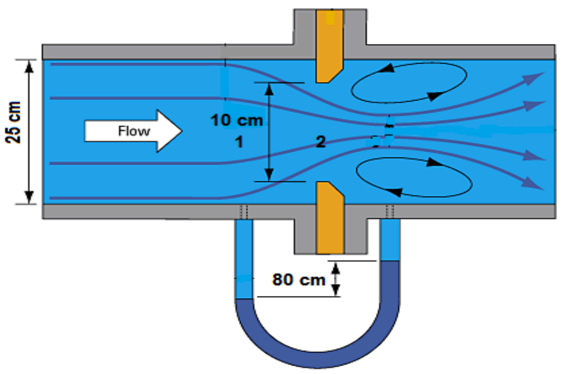
There is a vena contraction downstream of the orifice at a distance = (1/2) diameter of the orifice, say at section (3).

Introducing the effect of this contraction, Bernoulli's equation is applied for sections (1) and (3) obtaining another equation of orifice.

Example 7

An orifice meter has an orifice of 10 cm diameter and a coefficient of discharge of 0.65. It is fixed in a pipe of 25 cm diameter with flowing oil of 0.8 specific gravity. The pressure difference between the pipe and orifice is measured by a mercury manometer that gives a reading of 80 cm.

Determine the discharge.



Solution

For pipe,

$A_1 = (\pi\, d_1^2)/4 = (\pi * 25^2)/4 = 490.87\ cm^2$

For orifice,

$A_2 = (\pi\, d_2^2)/4 = (\pi * 10^2)/4 = 78.54\ cm^2$

$H = h_m\, [(\gamma_m / \gamma) - 1]$

$= 80\, [(13.6 / 0.8) - 1] = 1{,}280$ cm of oil

$$Q = \frac{490.87 x 78.54 \sqrt{2 x 981 x 1280}}{\sqrt{(490.87)^2 - (78.54)^2}} * 0.65 = 81957.8\ cm^3/s$$

4-6 Orifice under a Constant Head

The Orifice

It is a small opening in any vessel through which liquid flows. An orifice may be on the vertical side of the vessel or in its base. It may be rounded or sharp-edged. The main function of an orifice is measuring the discharge, as shown in Figure 12.

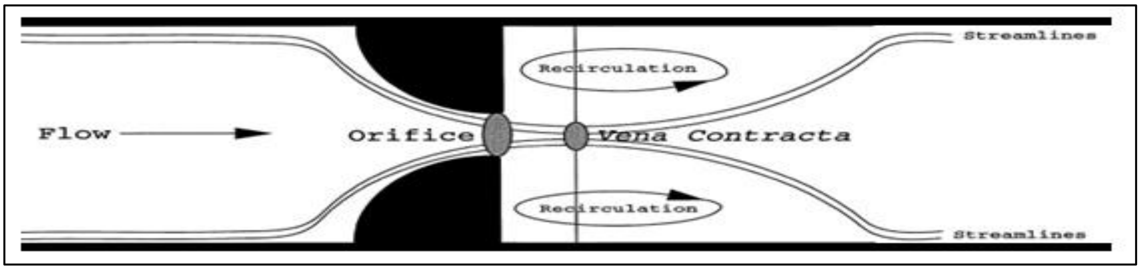


Figure 12. Orifice

https://www.youtube.com/watch?v=30kWRiOLvyw

Jet of Liquid

It is the continuous stream of liquid that flows out of the orifice.

Vena Contracta

As shown in Figure 13, fluid particles take a turn to enter the orifice from all directions. This consumes some energy from the flowing liquid. The liquid flowing out of the orifice (liquid jet) contracts as it is unable to make sharp turns. The maximum contraction (minimum cross-sectional area of the liquid jet) is found to be slightly downstream of the orifice. The section of maximum contraction is called vena contracta (section C-C in Figure 13).

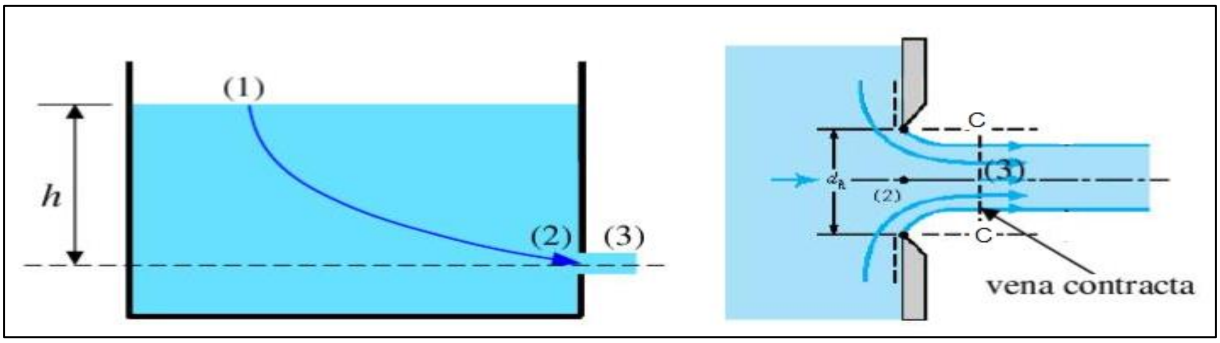


Figure 13. Orifice and Vena Contracta

https://issuu.com/bhagamravitejachowdary/docs/fluid-mechanics-notes-2/54

Applying Bernoulli's equation between the point (1) on the liquid surface and the point (3) at the centerline of vena contraction (section C-C in the figure),

$$h + 0 + 0 = 0 + 0 + (v^2/2g)$$

Where: h: Elevation of the liquid surface above vena contracta.

v: Velocity at vena contracta.

$$\boldsymbol{v = \sqrt{2\,g\,h}}$$

Coefficient of Contraction C_c

$$C_c = \frac{\text{Actual Area}}{\text{Theoretical Area}} = \frac{\text{Area of Jet at Vena Contraction}}{\text{Area of Orifice}} = A_c / A_o$$

$A_c = C_c\, A_o$ & $C_c = 0.61 - 0.69$

Coefficient of Velocity C_v

$$C_v = \frac{\text{Actual Velocity}}{\text{Theoretical Velocity}} = v_a / v$$

$v_a = C_v\, v$ & $C_v = 0.95 - 0.99$

Coefficient of Discharge C_d

$$C_d = \frac{\text{Actual Discharge}}{\text{Theoretical Discharge}} = Q_a / Q$$

$Q_a = C_d\, Q$

$Q = A_o\, v$

$Q_a = A_c\, v_a = (C_c\, A_o)\,(C_v\, v) = (C_c\, C_v)\,(A_o\, v)$

$$\mathbf{C_d = C_c\, C_v}$$

Example 8

The water is flowing through an orifice of 60 mm diameter with a head of 9 m. The coefficients of discharge and velocity are 0.6 and 0.9 respectively.

1- Calculate the actual discharge through the orifice.

2- Determine the actual velocity at vena contraction.

Solution

1- $A_o = \pi d^2 / 4 = (\pi\ 6^2) / 4 = 28.27\ cm^2$

$Q = A_o (2gH)^{1/2} = 28.27\ [2\ (981)\ (900)]^{1/2} = 37{,}566.14\ cm^3/s$

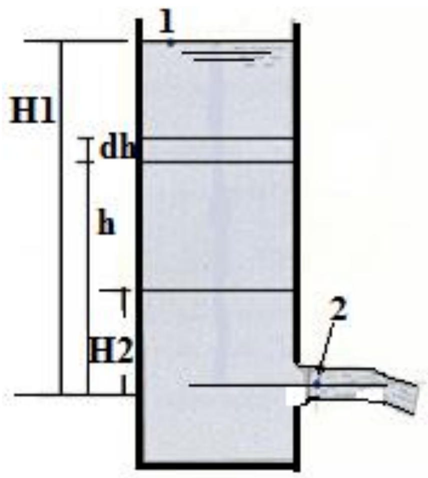
$Q_a = C_d\ Q = 0.6\ (37566.14) = 22{,}539.68\ cm^3/s$

2- $v = (2gH)^{1/2} = [2\ (981)\ (900)]^{1/2} = 1{,}328.83\ cm/s$

$v_a = C_v\ v = 0.9\ (1328.83) = 1{,}195.95\ cm/s$

4-7 Orifice under a Varying Head

Assume a tank, as shown in the figure, of a cross-sectional area A_T contains a liquid that is flowing through an orifice. During period T the liquid level decreased from H_1 to H_2.



At some instant, the liquid height is (h+dh) above the orifice. During a small interval of time dt, the head decreases by a small amount of dh.

The actual discharge of liquid flowing out of the orifice is dQ,

$$dQ = C_d\ A_o\ (2\ g\ h)^{1/2}$$

The volume of liquid flowing out of the orifice is dV,

$dV = dQ\ dt = C_d\ A_o\ (2\ g\ h)^{1/2}\ dt$

Also, concerning the tank, the liquid volume in it decreased during the time interval dt by the amount dV,

$dV = -\ A_T\ dh$

Thus, $C_d\ A_o\ (2\ g\ h)^{1/2}\ dt = -\ A_T\ dh$

$dt = -\ A_T\ dh\ /\ C_d\ A_o\ (2\ g\ h)^{1/2}$

$dt = -\ A_T\ dh\ (h)^{-1/2}\ /\ C_d\ A_o\ (2\ g)^{1/2}$

The total time T for decreasing liquid level from H_1 to H_2 is:

$$T = \frac{-\ A_T}{C_d\ A_0\ \sqrt{2\ g}}\ h^{\frac{-1}{2}}\ dh$$

$$T = \frac{-\ 2\ A_T\ (\sqrt{H_2} - \sqrt{H_1})}{C_d\ A_0\ \sqrt{2\ g}}$$

Minus sign may be eliminated by using (H_1 - H_2), where $H_1 > H_2$.

$$\boldsymbol{T = \frac{2\ A_T\ (\sqrt{H_1} - \sqrt{H_2})}{C_d\ A_0\ \sqrt{2\ g}}}$$

Applications

- This equation can be used to determine the time T required to reduce the liquid surface from one level to another.

- Also, it can be applied to detect the time T required to empty any tank containing a liquid. In this case, the liquid level decreases from H_1 to zero.

Example 9

A swimming pool 10 m long and 6 m wide holds water to a depth of 1.25 m. The water is discharged from the pool through an orifice at its bottom. The area of the orifice is 0.23 m^2, and the coefficient of discharge is 0.62.

Determine the time required to empty the pool.

Solution

$A_T = 10 * 6 = 60\ m^2$

$A_o = 0.23\ m^2$

$H_1 = 1.25$ m & $H_2 = 0$

$$T = \frac{2\ A_T\ (\sqrt{H_1} - \sqrt{H_2})}{C_d\ A_0\ \sqrt{2\ g}} = \frac{2 * 60\ (\sqrt{1.25} - 0)}{0.62 * 0.23\ \sqrt{2 * 9.81}} = 212\ s$$

Chapter 5

MOMENTUM ANALYSIS OF FLOW SYSTEMS

5-1 Momentum

Newton's laws are relations between the motions of bodies and the acting forces.

- Newton's first law states that a body at rest remains at rest, and a body in motion remains in motion at the same velocity in a straight path when the net force acting on it is zero.
- Newton's second law states that the acceleration of a body is proportional to the net force acting on it and is inversely proportional to its mass.
- Newton's third law states that when a body exerts a force on a second body, the second body exerts an equal and opposite force on the first.

For a rigid body of mass m, Newton's second law is expressed as:

$$F = m * a$$

where F is the net force acting on the body and a is the acceleration of the body under the influence of F.

Linear momentum or just the momentum of any moving body (such as a flowing fluid particle) is the quantity of motion. It is the product of its mass and velocity.

Momentum = Mass * Velocity = m * v

Units: *SI system:* kg.m/s

$$F = m * a = m \frac{dv}{dt} = \frac{d(mv)}{dt}$$

The rate of change of the momentum of a body is equal to the net force acting on the body and takes place in the direction of the force. This statement is more in line with Newton's original statement of the second law.

So, in fluid mechanics, Newton's second law is usually referred to as the linear momentum equation.

As the velocity, the momentum is a vector quantity.

Examples:

- The lift force on an aircraft is exerted by the air moving over the wing.
- A jet of water from a hose exerts a force on whatever it hits.

5-2 Momentum for One-Dimensional Flow

For a steady and non-uniform flow in the stream tube shown in Figure 1:

Mass entering the streamtube = Density * Volume = $\rho * A_1 (v_1 t)$

Momentum entering the streamtube = Mass*Velocity = $\rho A_1 (v_1 t) * v_1$

Similarly,

Mass leaving the streamtube = Density * Volume = $\rho * A_2 (v_2 t)$

Momentum leaving the streamtube = Mass*Velocity = $\rho A_2 (v_2 t) * v_2$

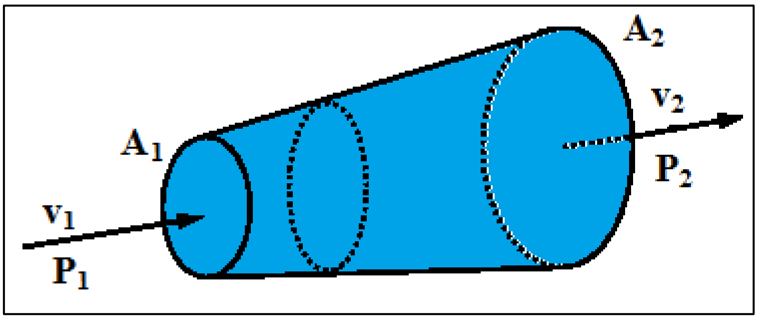


Figure 1. Momentum for One-Dimensional Flow

The resultant force on the fluid = Rate of change of momentum

$F = [\rho\ A_2\ (v_2\ t) * v_2 - \rho\ A_1\ (v_1\ t) * v_1] / t$ & $Q = A_2\ v_2 = A_1\ v_1$

$F = [\rho\ Q * v_2 - \rho\ Q * v_1]$

∴ $F = \rho\ Q\ (v2 - v1)$

The resultant force is acting in the direction of the flow of the fluid.

5-3 **Momentum for Two-Dimensional Flow**

For a steady and non-uniform flow in the stream tube shown in Figure 2,

In x – direction,

F_x = Rate of change of momentum in x - direction

$F_x = \rho\ Q\ (v_{2x} - v_{1x})$

In y – direction,

F_y = Rate of change of momentum in y - direction

$F_y = \rho\ Q\ (v_{2y} - v_{1y})$

The resultant force: $F = \sqrt{Fx^2 + Fy^2}$

F is inclined with an angle ∝ to the x-axis: $\propto = \tan^{-1}(F_y / F_x)$

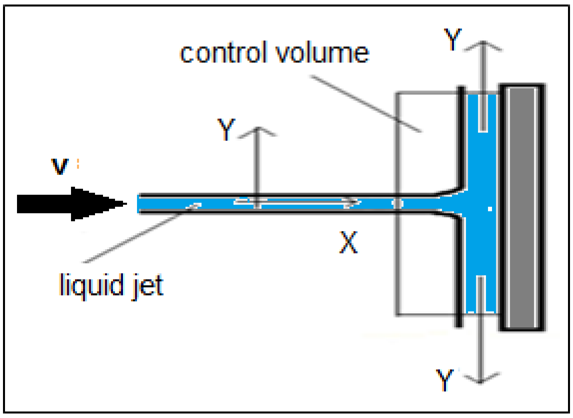


Figure 2. Momentum for Two-Dimensional Flow

5-4 Momentum for Three-Dimensional Flow

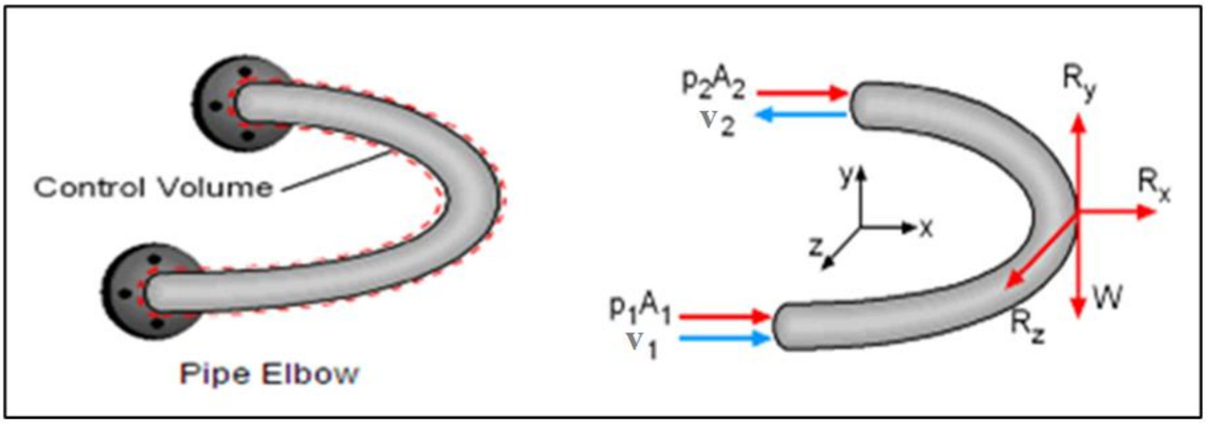


Figure 3. Momentum for Three-Dimensional Flow

In general, $F_T = F_B + F_W + F_P = \rho\, Q\, (v_{out} - v_{in})$

F_T : The total force.

F_B : The force exerted on the fluid by the surrounding boundary.

F_W : The force exerted on the fluid by gravity (the weight).

F_P : The force exerted on the fluid by the pressure.

According to Newton's third law, the fluid will exert an equal and opposite reaction.

The reaction or the force exerted by the fluid on the surrounding boundary is equal to and opposite to (F_B).

$R = -F_B$

Example 1 *(Force Applied by a Liquid Jet on a Flat Plate)*

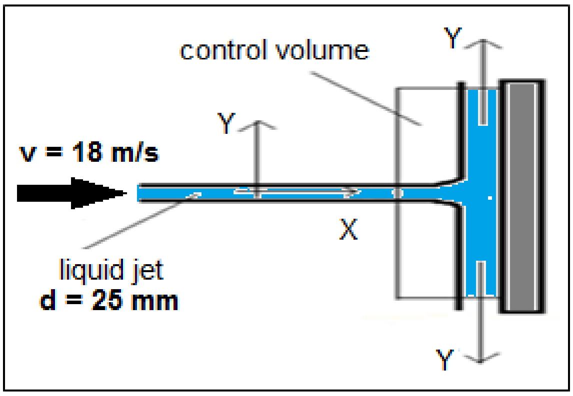
A flat fixed plate is hit normally by a jet of water 25 mm in diameter with a velocity of 18 m/s, as shown in the figure.

Find the force on the plate.

Solution

In x-direction:

$F_{Tx} = F_B + F_W + F_P = \rho\ Q\ (v_{out} - v_{in})_x$



F_B : The force exerted on the fluid by the surrounding boundary (the plate).

F_W : The gravity force. $F_W = 0$ (Negligible weight of water)

F_P : The pressure force. $F_P = 0$ (Atmospheric pressure)

$F_B + 0 + 0 = \rho\ Q\ (v_{out} - v_{in})_x$

$F_B = 1{,}000*18*(\pi 0.025^2/4)*(0 - 18) = - 162$ N *in – ve x - direction*

$\therefore$ The force on the plate $R = - F_B = 162$ N *in + ve x - direction*

In y-direction:

$F_{Ty} = F_B + F_W + F_P = \rho\ Q\ (v_{out} - v_{in})_y$

$(v_{out} = v_{in})_y$

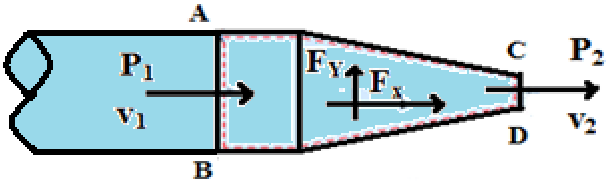
$\therefore$ No force in the y-direction.

Example 2

A nozzle is connected to a hose. At the entry section, the pressure is 250 kPa and the diameter is 25 mm. At the exit section, the diameter is 10 mm. The flow rate is 1 lit/s.

Determine the force required to hold the nozzle.

Solution



$q = 1$ lit/s $= 0.001\ m^3/s$

At entry: $P_1 = 250$ k Pa $d_1 = 25$ mm

At exit: $P_2 = 1$ atm $= 0$ $d_2 = 10$ mm

In x – direction:

$v_1 = Q / A_1 = 0.001 / (\pi\ (0.025)^2/4) = 2.04$ m/s

$v_2 = Q / A_2 = 0.001 / (\pi\ (0.01)^2/4) = 12.73$ m/s

$F_{Tx} = F_B + F_W + F_P = \rho\ Q\ (v_{out} - v_{in})_x$

$F_B + 0 + (P_1\ A_1 - 0) = \rho\ Q\ (v_2 - v_1)_x$

$F_B = -\ [(250{,}000 * (\pi\ 0.025^2/4)] + 1{,}000 * 0.001 * (12.73 - 2.04)$

$\therefore F_B = -\ 112$ N *in – ve x - direction*

Thus, the required holding force is $R = -F_B = 112$ N *in + ve x - direction*

In y – direction:

$v_{out\ Y} = v_{in\ Y} = 0$

$\therefore$ No force in the y-direction.

Chapter 6

FLOW IN PIPES

6-1 Flow in Pipes

Pipes are closed conduits (circular cross-sections in general) that are used for carrying fluids.

Circular pipes can withstand large pressure differences between the inside and the outside without undergoing any significant distortion, but noncircular pipes cannot, as shown in Figure 1.

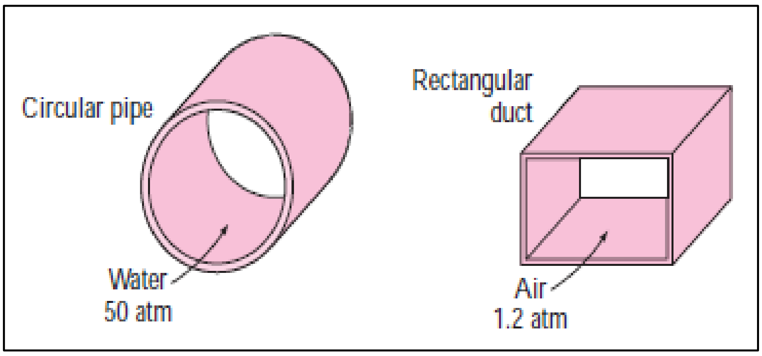


Figure 1. Types of Pipes

6-2 Velocity in Pipes

The fluid velocity in a pipe, changes from zero at the surface because of the no-slip condition to a maximum at the pipe center, as shown in Figure 2.

In fluid flow, it is convenient to work with an average velocity V_{avg}, which remains constant in incompressible flow when the cross-sectional area of the pipe is constant.

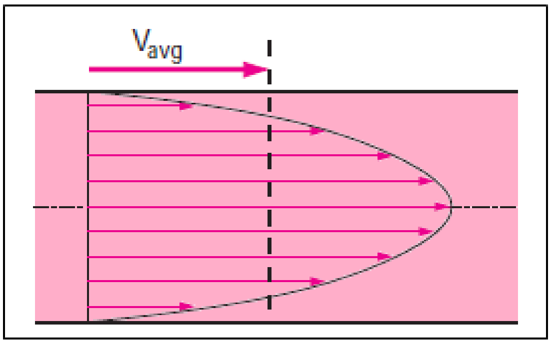


Figure 2. Velocity in Pipes

The fluid flow in a pipe reveals that the flow is streamlined at low velocities but turns messy as the velocity increases above a critical value, as shown in Figure 3.

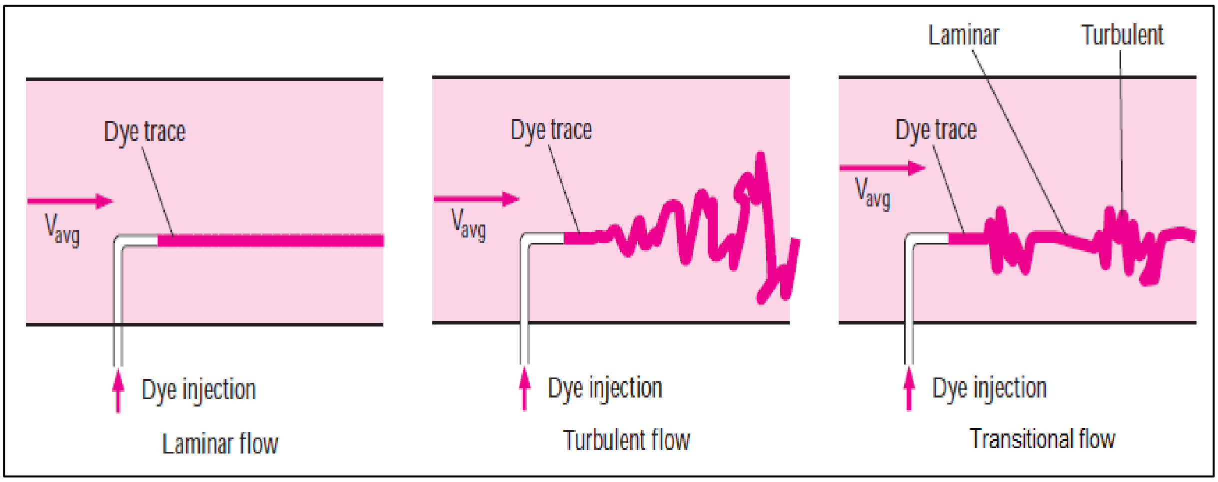


Figure 3. Laminar and Turbulent Flow in Pipes

- The flow regime in the first case is said to be laminar, characterized by smooth streamlines and highly ordered motion.
- While the flow regime in the second case is turbulent, where it is characterized by velocity fluctuations and highly disordered motion.
- The transition from laminar to turbulent flow does not occur suddenly; rather, it occurs over some regions in which the flow fluctuates between laminar and turbulent flows before it becomes fully turbulent.

- Most flows encountered in practice are turbulent. Laminar flow is encountered when highly viscous fluids such as oils flow in small pipes or narrow passages.

The flow regime depends mainly on the ratio of inertial forces to viscous forces in the fluid. This ratio is called the Reynolds number and is expressed for the flow in a circular pipe as:

$$R_e = \frac{Inertia\ force}{Viscousity\ force} = \frac{v_{av}\ d}{\nu} = \frac{\rho\ v_{av}\ d}{\mu}$$

where V_{av} = average flow velocity (m/s), d = characteristic length of the geometry (diameter in this case, in m), and ν = μ/r = kinematic viscosity of the fluid (m^2/s).

Note that the Reynolds number is a dimensionless quantity.

Re ≤ 2,300	laminar flow
2,300 < Re < 4,000	transitional flow
Re ≥ 4,000	turbulent flow

The velocity profile for laminar flow in a pipe is parabolic with a maximum at the centerline and zero at the pipe wall, as shown in Figure 4. Also, the axial velocity v is positive for any r, and thus the axial pressure gradient dP/dx must be negative (i.e., pressure must decrease in the flow direction because of viscous effects).

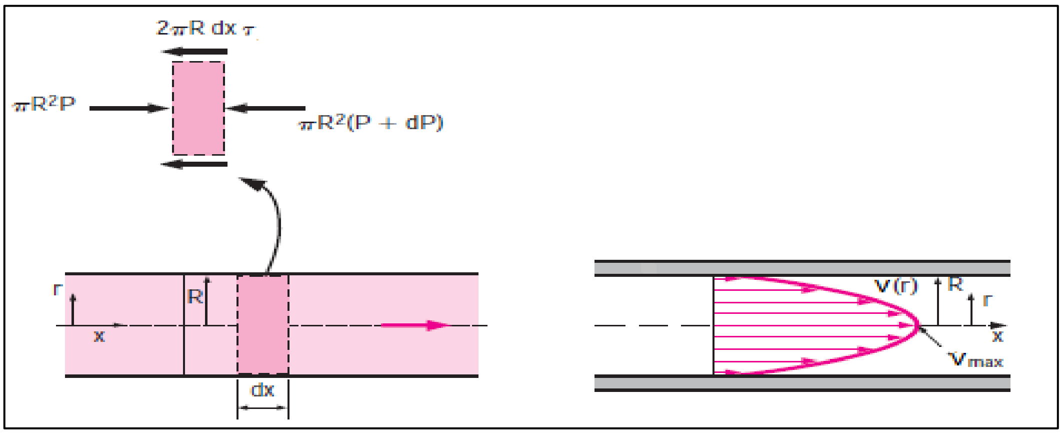


Figure 4. Free-body Diagram of a Fluid Element & Velocity Profile

For laminar flow:

$\sum F = 0$

$\pi R^2 P - \pi R^2 (P + dP) - 2 \pi R \, dx \, \tau = 0$

$$\frac{dP}{dx} = -\frac{2\tau}{R}$$

$$v(r) = 2\, v_{av}\, (1 - \frac{r^2}{R^2})$$

$r = 0$ $\qquad v_{max} = 2\, v_{av}$ $\qquad v_{av} = Q / A = \text{Constant}$

For turbulent flow:

$$v(r) = 1.23\, v_{av}\, (1 - \frac{r^2}{R^2})$$

$r = 0$ $\qquad v_{max} = 1.23\, v_{av}$ $\qquad v_{av} = Q / A = \text{Constant}$

6-3 Head Losses in Pipes

There are many types of losses of the head for flowing liquids such as friction, inlet, and outlet losses, as shown in Figure 5.

Main Losses	Secondary Losses
Friction.	Inlet.
	Outlet.
	Changing area.
	Changing direction.
	Fittings (e.g. valves).

Figure 5. Losses in Pipes

Main Losses Due to Friction

The major loss in pipe flow is due to the frictional resistance of the pipe, which depends on the inside roughness of the pipe, as shown in Figure 6. The common formula for calculating the loss of head due to friction is the Darcy-Weisbach equation.

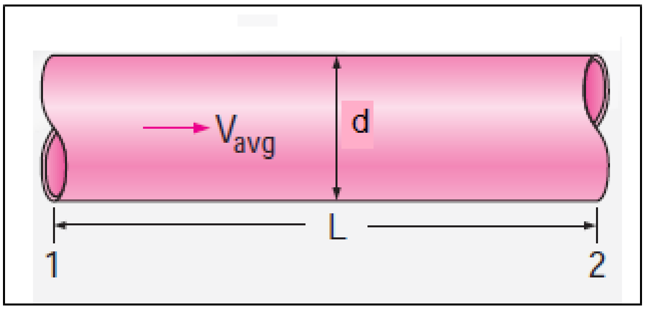


Figure 6. Friction Loss in Pipes

$$h_f = f \frac{L}{d} \frac{v_{av}^2}{2g} \qquad h_f = f \frac{L}{d} \frac{Q^2}{2gA^2} = \frac{8 \, f \, L \, Q^2}{\pi^2 \, g \, d^5}$$

Where h_f = energy head loss due to friction, f = friction factor, L = length of pipe, d = diameter of pipe, and v_{av} = mean (average) velocity, Q = discharge, A = cross-sectional area.

The friction factor in fully developed turbulent pipe flow depends on the Reynolds number and the relative roughness ε/D, which is the ratio of the mean height of roughness of the pipe to the pipe diameter.

The available data for transition and turbulent flow in smooth, as well as rough pipes, were combined into the following Colebrook equation:

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon/d}{3.7} + \frac{2.51}{R_e\sqrt{f}}\right)$$

Note: *In non-American practice and references,* $f_{American} = 4f = \lambda$

The now-famous Moody chart, as shown in Figure 7, presents the Darcy friction factor for pipe flow as a function of the Reynolds number and ε/D over a wide range. Although it is developed for circular pipes, it can also be used for noncircular pipes by replacing the diameter with the hydraulic diameter.

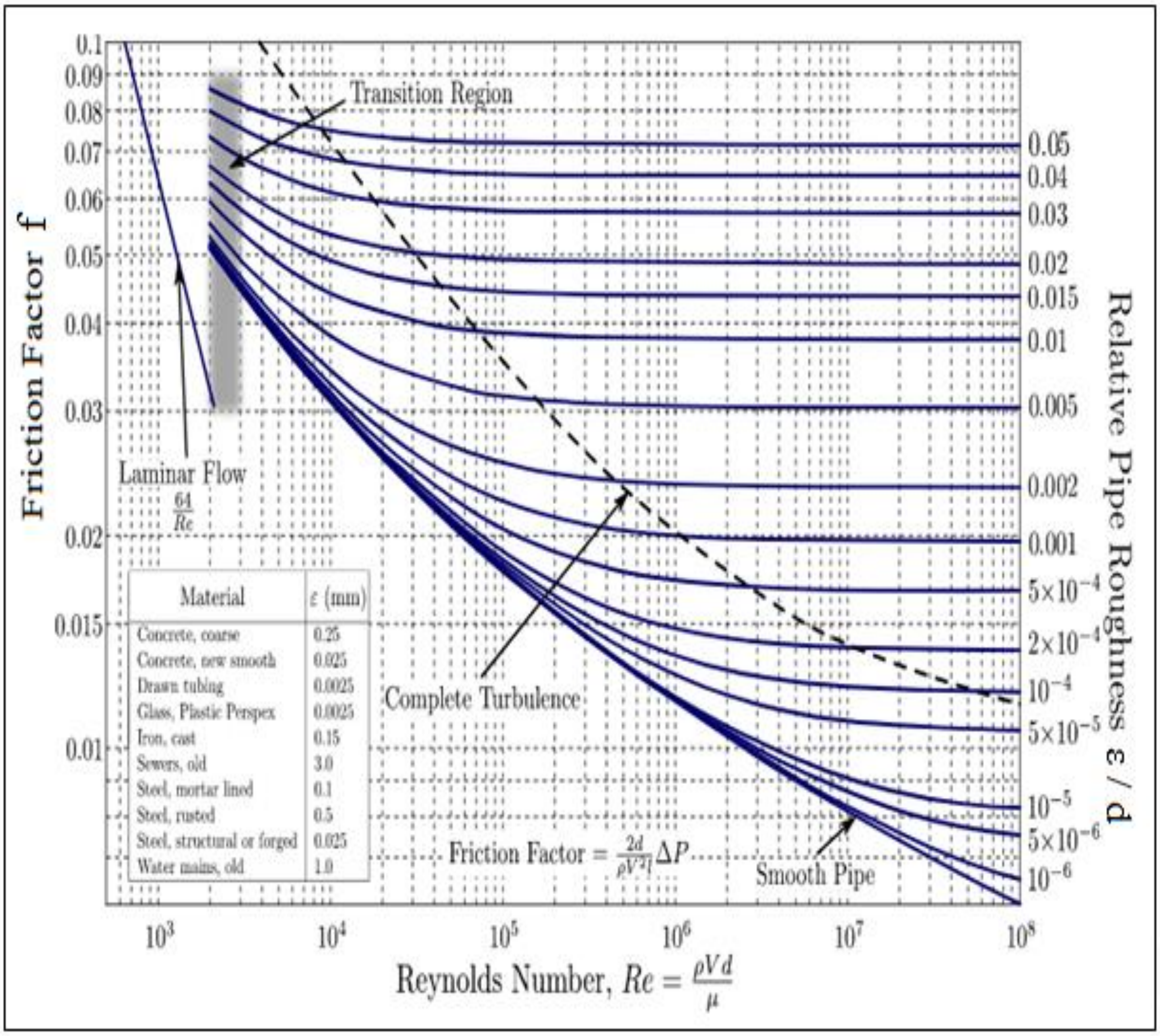


Figure 7. Moody Chart

- For laminar flow, the friction factor is a function of the Reynolds number only and is independent of the roughness of the pipe surface.

- ➢ The friction factor is a minimum for a smooth pipe (but still not zero) and increases with roughness.
- ➢ The transition region from the laminar to the turbulent regime (2300 < Re < 4000) is indicated by the shaded area in the Moody chart. The flow in this region may be laminar or turbulent, or it may alternate between them.
- ➢ At very large Reynolds numbers (to the right of the dashed line on the chart), the friction factor curves corresponding to relative roughness curves are nearly horizontal, and thus the friction factors are independent of the Reynolds number, as shown in Figure 8.

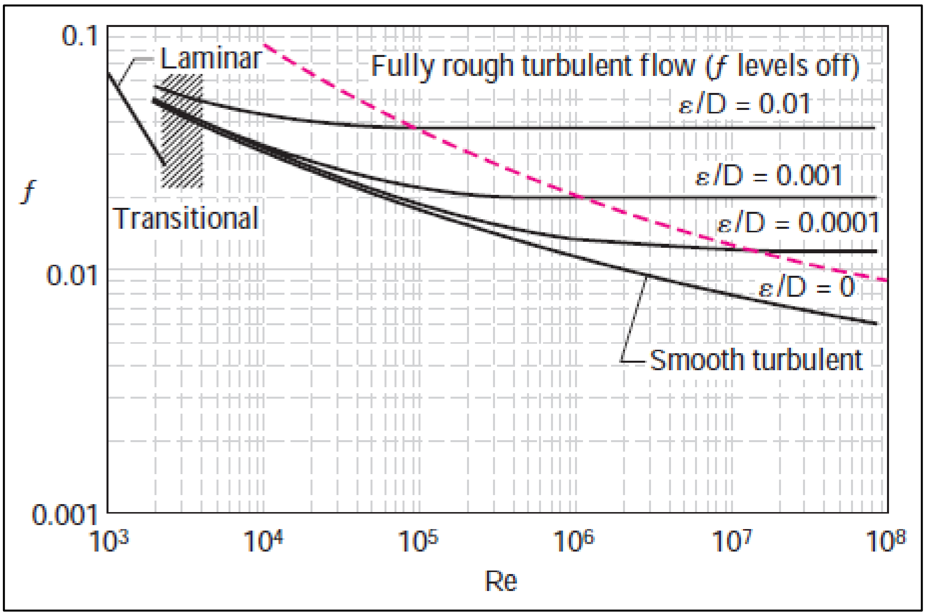


Figure 8. Horizontal f Curves for Large R_e

Equivalent roughness values for some commercial pipes are shown in Table 1 (as well as on the Moody chart).

However, it should be noted that these values are for new pipes, and the relative roughness of pipes may increase with use because of corrosion, scale buildup, and precipitation.

Table 1. Roughness (ε) Values for New Commercial Pipes

Material	Roughness, ε	
	ft	mm
Glass	0	0
Plastic	0	0
Concrete	0.003 – 0.03	0.9 - 9
Cast Iron	0.00085	0.26
Commercial Steel	0.00015	0.045
Stainless Steel	0.000007	0.002

Example 1

A pipe 1 m in diameter and 15 km long transmits water at a velocity of 1 m/sec. The friction coefficient of the pipe is 0.02.

Calculate head loss due to friction.

Solution

$$h_f = f\ \frac{L}{d}\ \frac{v_{av}^2}{2g} = \frac{0.02 * 15{,}000 * 1^2}{1 * 2 * 9.81} = 15.3\ m$$

Example 2

Water flows in a steel pipe (d = 40 mm, ε = 0.045×10^{-3} m, μ = 0.001 k/m.s) with a rate of 1 L/s.

Determine the friction factor and the head loss due to friction per meter length of the pipe using the Moody chart.

Solution

v = Q / A = 0.001 / (π $(0.04)^2$/4) = 0.796 m/s

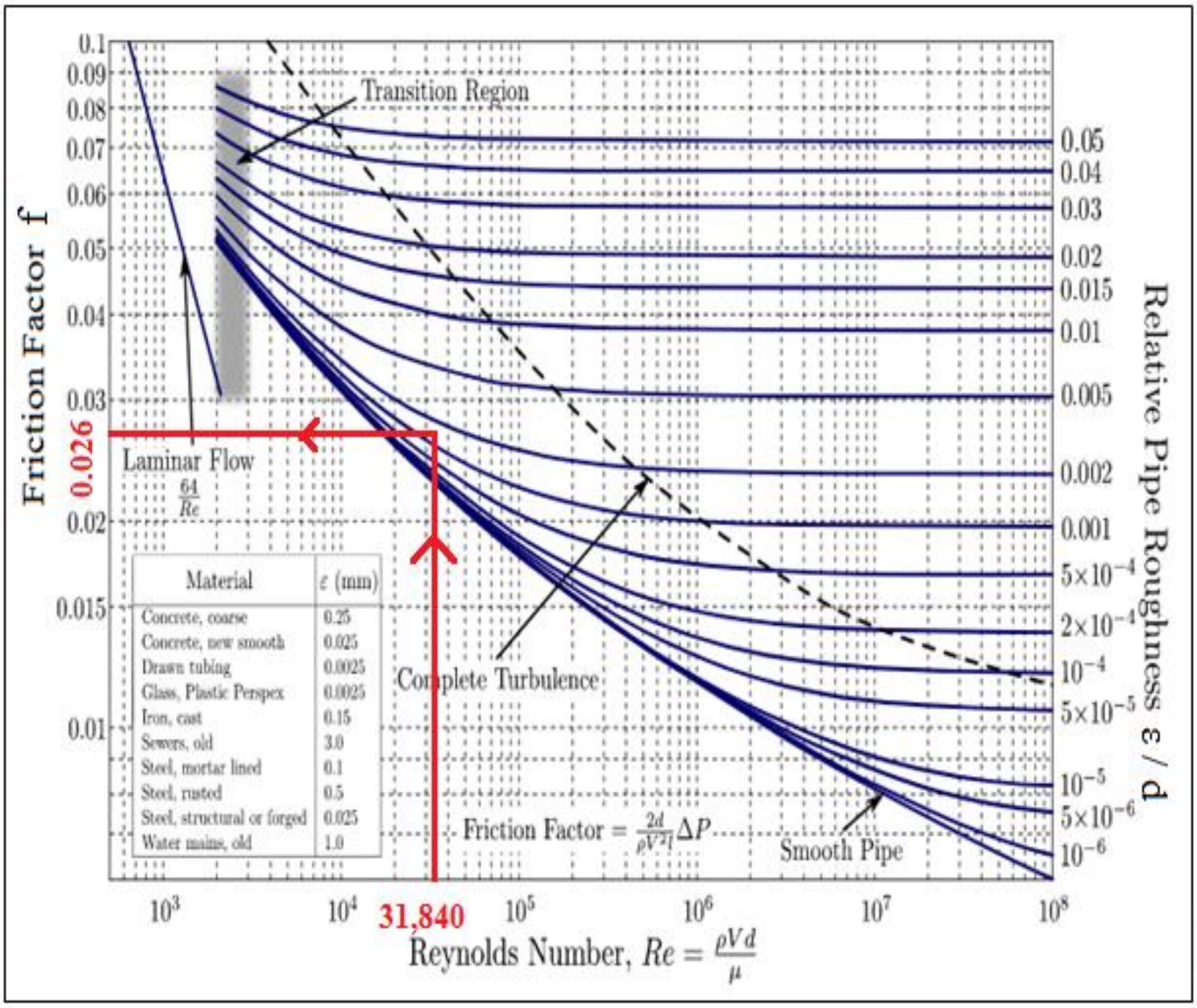
R_e = ρ v d / μ = (1,000*0.796*0.04) / 0.001 = 31,840 > 4,000 ∴ Turbulent flow.

From Moody chart:

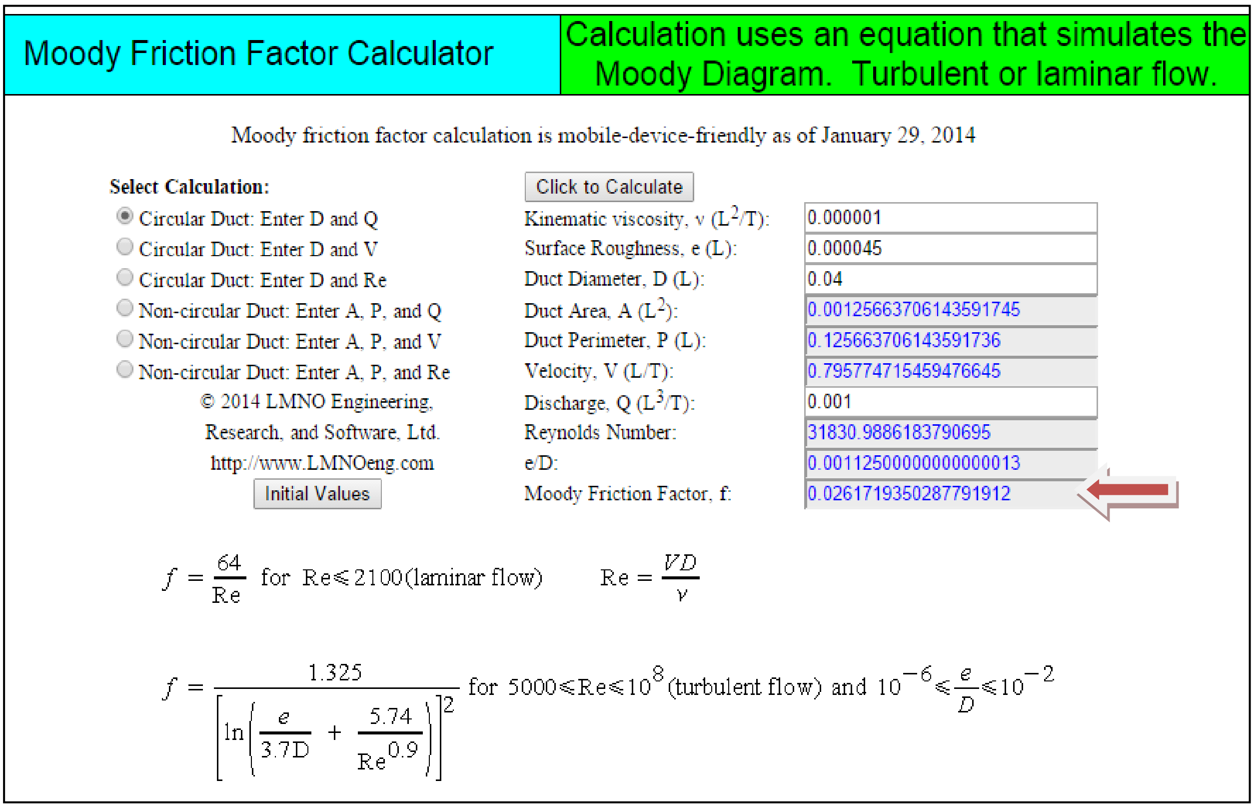
ε /d = 0.045x10^{-3} / 0.04 = 0.0011 & R_e = 31,840

∴From the chart, f = 0.026

$$h_f = f \frac{L}{d} \frac{v_{av}^2}{2g} = \frac{0.026 * 1 * 0.796^2}{1 * 2 * 9.81} = 0.021\, m\, /\, m\ of\ pipe$$



Another Solution



https://www.lmnoeng.com/moody.php

Secondary Losses

Any change in a pipe (in direction, diameter, having a valve, or other fittings) will cause a loss of energy due to the disturbance in the flow.

$$h_L = K_L (v^2 / 2g)$$

Where h_L = the head loss, K_L = the loss coefficient, and v = the velocity at the entry to the fitting.

- The loss coefficient for a pipe inlet is shown in Figure 9, including the effect of rounding a pipe inlet on the loss coefficient.
- The loss coefficient for a pipe outlet is shown in Figure 10.

- The loss coefficients for sudden and gradual expansion and contraction are shown in Figures 11 and 12.
- The loss coefficient for a pipe bend (elbow) is shown in Figure 13.
- The loss coefficients for different valves are shown in Table 2.
- Figure 14 shows the graphical representation of flow contraction and the associated head loss at a sharp-edged pipe inlet.

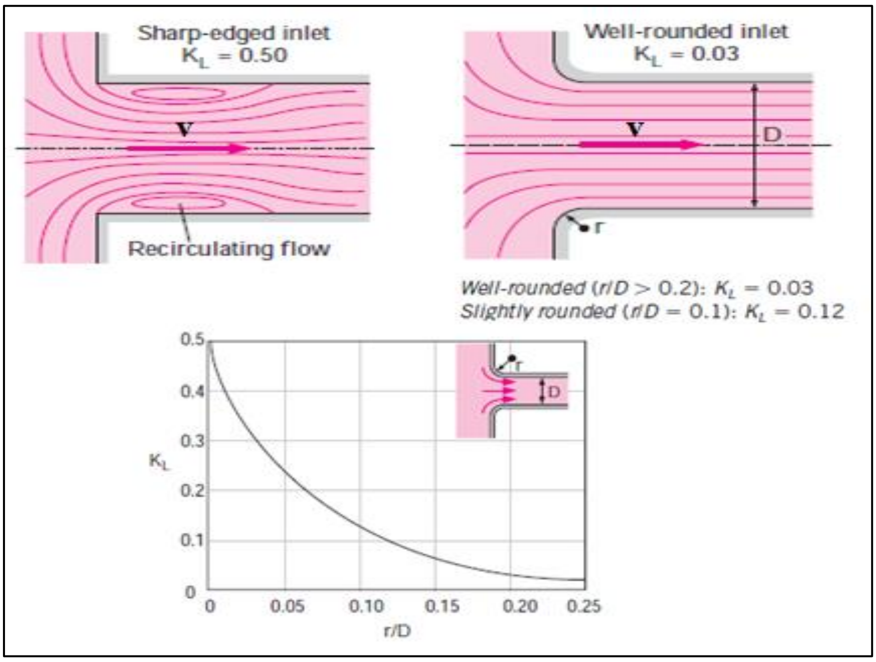


Figure 9. K_L for a Pipe Inlet

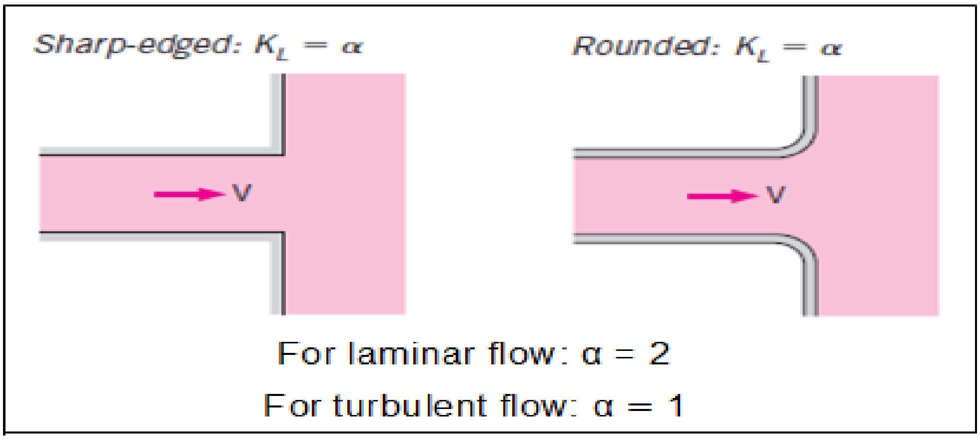


Figure 10. K_L for a Pipe Outlet

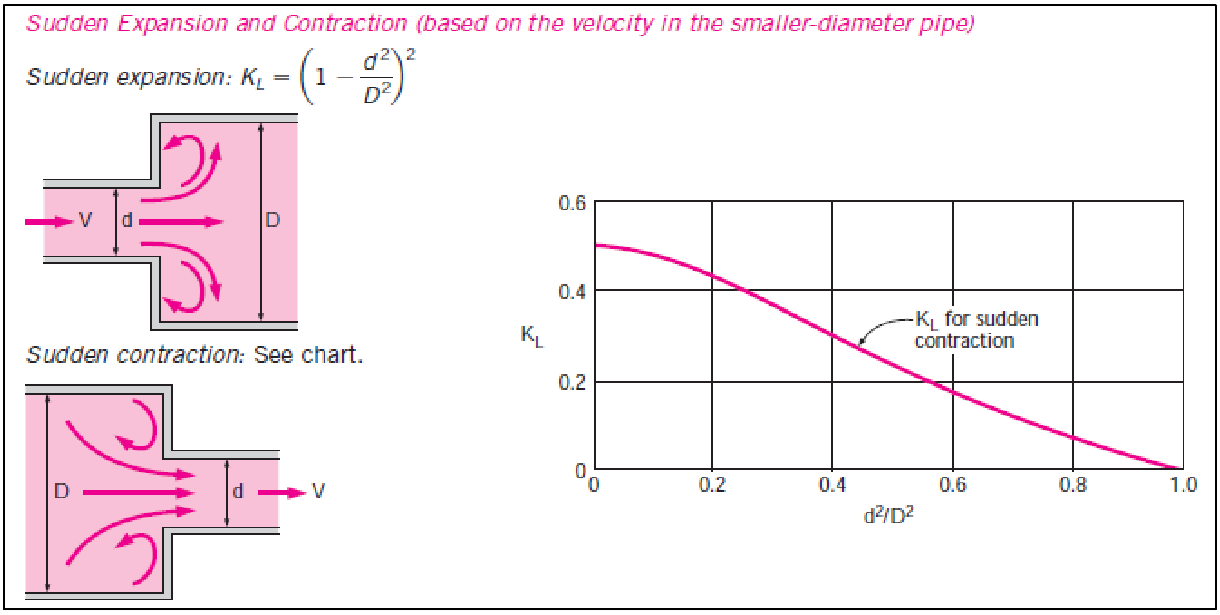


Figure 11. K_L for Sudden Expansion and Contraction

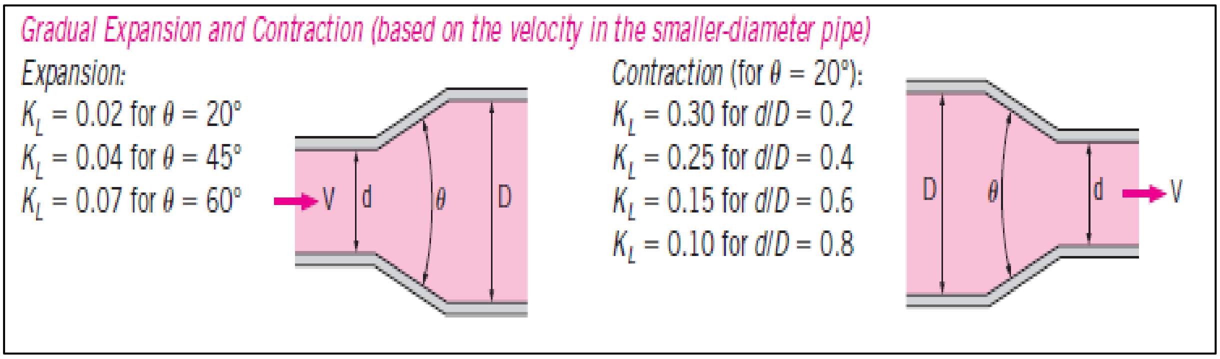


Figure 12. K_L for Gradual Expansion and Contraction

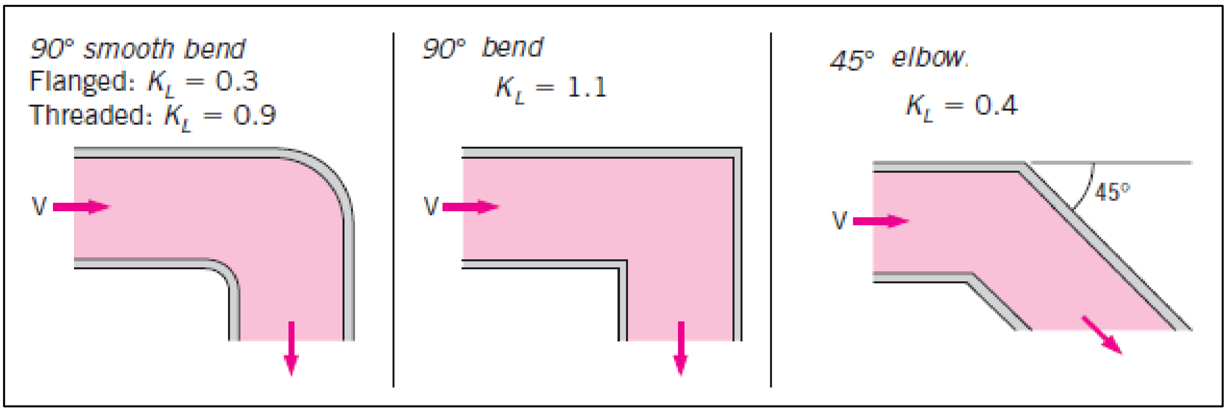


Figure 13. K_L for Bend (Elbow)

Table 2. Loss Coefficients, K_L, for Different Valves

Valve	Notes	Loss Coefficients, KL
Globe valve	Fully open	**10**
Angle valve	Fully open	**5**
Ball valve	Fully open	**0.05**
Swing check valve	---	**2**
Gate valve	Fully open	**0.2**
	¼ Closed	**0.3**
	½ Closed	**2.1**
	¾ Closed	**17**

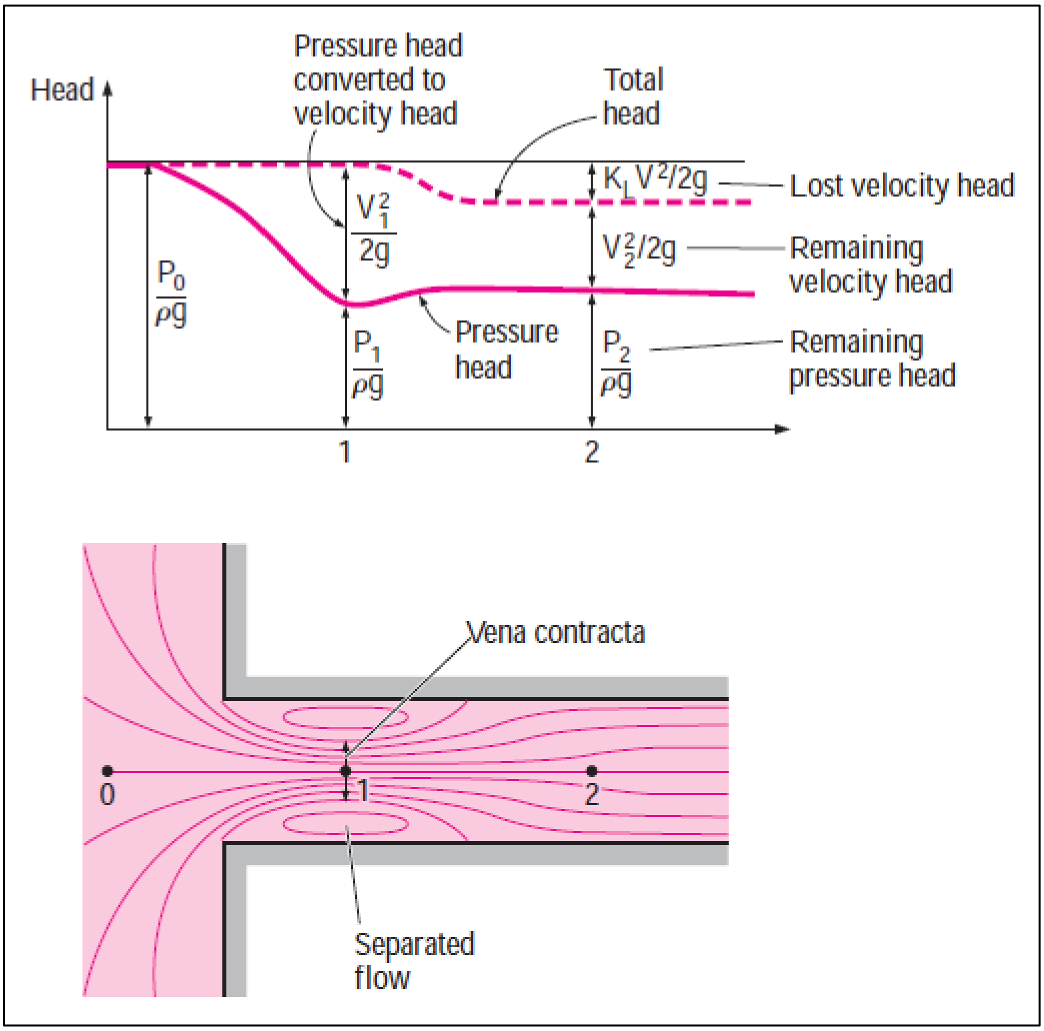


Figure 14. Graphical Representation of Flow Contraction and Sharp Inlet

From Figure 14, a sharp-edged inlet causes half of the velocity head to be lost as the fluid enters the pipe. This is because the fluid cannot make sharp 90° turns easily, especially at high velocities. As a result, the flow separates at the corners, and the flow is constricted into the vena contracta region formed in the midsection of the pipe. So, a sharp-edged inlet acts like a flow constriction.

The losses are usually much greater in the case of sudden expansion than those of contraction (or wide-angle expansion) because of flow separation.

By combining the conservation of mass, momentum, and energy equations, the loss coefficient for the case of sudden expansion is approximated as:

$$K_L = (1 - \frac{A_{small}}{A_{large}})^2$$

Where A_{small} and A_{large} are the cross-sectional areas of the small and large pipes, respectively.

Note that $K_L = 0$ when there is no area change ($A_{small} = A_{large}$) and $K_L = 1$ when a pipe discharges into a reservoir ($A_{large} >> A_{small}$).

No such relationship exists for a sudden contraction, and the K_L values in that case can be read from Figure 11.

6-4 <u>Pipes in Series and Parallel</u>

When the pipes are connected in series, as shown in Figure 15, the flow rate through the entire system remains constant regardless of the diameters of the individual pipes in the system. This is because of the conservation of mass principle for steady incompressible flow.

The total head loss, in this case, is equal to the sum of the head losses in individual pipes in the system, including the minor losses.

$H_{LT} = H_{LA} + H_{LB} + \ldots\ldots$ **Q = Constant**

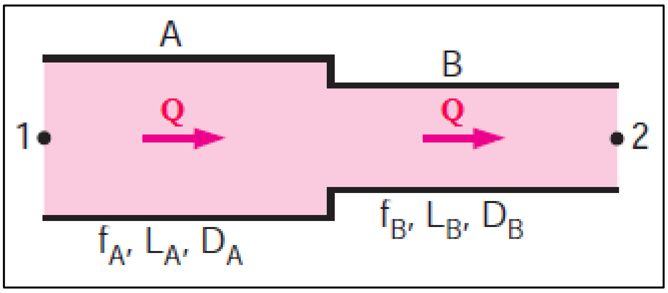


Figure 15. Pipes in Series

For pipes in parallel, the head loss is the same in each pipe, and the total flow rate is the sum of the flow rates in individual pipes, as shown in Figure 16.

$Q = Q_1 + Q_2 + \ldots\ldots$ **$H_{LT} = H_{L1} = H_{L2} = \text{Constant}$**

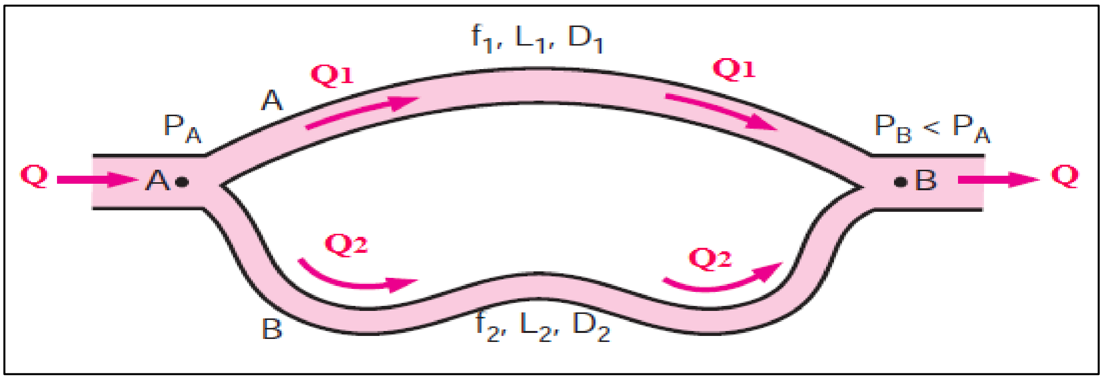


Figure 16. Pipes in Parallel

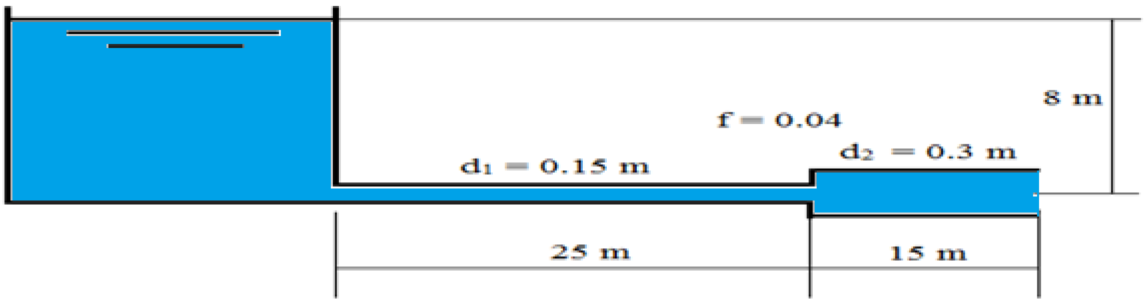
The expansion or contraction losses at connections are considered to belong to the smaller-diameter pipe since the expansion and contraction loss coefficients are defined based on the average velocity in the smaller-diameter pipe.

Example 3

A pipe, 40 m long, is connected to a water tank at one end and flows freely in the atmosphere at the other end. The diameter of the pipe is 15 cm for the first 25 m from the tank, and then the diameter is suddenly enlarged to 30 cm. Height of water in the tank is 8 m above the center of the pipe. The friction coefficient is 0.04.

Determine the discharge neglecting minor losses.

Solution



Loss due to friction, $h_T = h_{f1} + h_{f2}$

$$h_f = \frac{8\, f\, L\, Q^2}{\pi^2\, g\, d^5}$$

$$h_T = Q^2\left(\frac{8\, f\, L_1}{\pi^2\, g\, d_1^5} + \frac{8\, f\, L_2}{\pi^2\, g\, d_2^5}\right)$$

$$h_T = Q^2\left(\frac{8 * 0.04 * 25}{\pi^2 * g * 0.15^5} + \frac{8 * 0.04 * 15}{\pi^2 * g * 0.3^5}\right)$$

$\therefore Q = 0.087\ m^3/sec$

Example 4

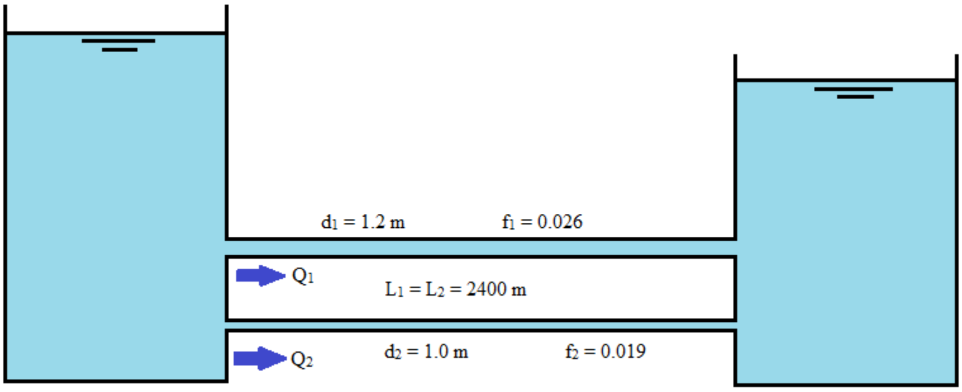
Two pipes are connected in parallel between two reservoirs that have a difference in levels of 3.5 m. The length, diameter, and Darcy's coefficient are 2400 m, 1.2 m, and 0.026 for the first pipe and 2400 m, 1 m, and 0.019 for the second pipe, respectively.

Calculate the total discharge between the two reservoirs.

Solution

Pipe 1: $L_1 = 2400$ m $d_1 = 1.2$ m $f_1 = 0.026$

Pipe 2: $L_2 = 2400$ m $d_2 = 1.0$ m $f_2 = 0.019$



$$h_L = \frac{8\, f_1\, L\, Q_1^2}{\pi^2\, g\, d_1^5} = \frac{8\, f_2\, L\, Q_2^2}{\pi^2\, g\, d_2^5}$$

$$3.5 = \frac{8\, f_1\, L\, Q_1^2}{\pi^2\, g\, d_1^5}$$

$Q_1 = 1.29\ m^3/s$

$$3.5 = \frac{8\, f_2\, L\, Q_2^2}{\pi^2\, g\, d_2^5}$$

$Q_2 = 0.96\ m^3/s$

$\therefore Q = Q_1 + Q_2 = 1.29 + 0.96 = 2.25\ m^3/s$

Example 5

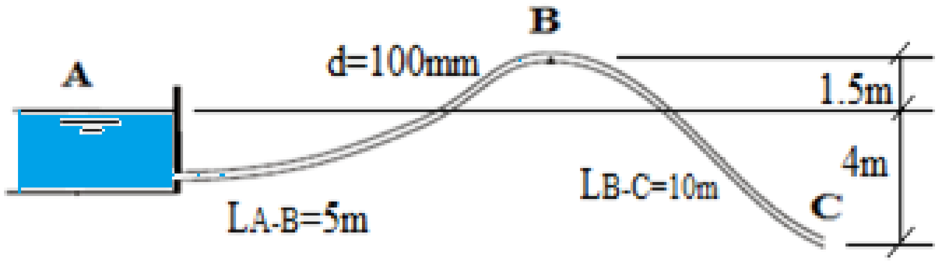
A pipe transmits water from tank A to point C that is lower than the water level in the tank by 4 m. The pipe is 100 mm in diameter and 15 m long.

The highest point on pipe B is 1.5 m above the water level in the tank and 5 m long from the tank. The friction coefficient is 0.08, with a sharp inlet and outlet to the pipe.

1) Determine the velocity of water leaving the pipe at point C.

2) Calculate the pressure in the pipe at point B.

Solution



1) *Applying Bernoulli's equation between A and C,*

K_L due to inlet (tank exit, *from Table 2*) = 0.5

K_L due to exit into the air without contraction = 0

$Z_A + P_A/\gamma + (v^2{}_A/2g) = Z_C + P_C/\gamma + (v^2{}_C/2g) + 0.5(v^2{}_C/2g) + h_f$

$4 + 0 + 0 = 0 + 0 + (v^2{}_C/2g) + 0.5(v^2{}_C/2g) + (fL/d)*(v^2{}_C/2g)$

$4 = (v^2{}_C/2g)*\{1 + 0.5 + (0.08*15)/0.1\}$

$\therefore v_C = 1.26$ m/s

2) *Applying Bernoulli's equation between A and B,*

$Z_A + P_A/\rho g + (v^2{}_A/2g) = Z_B + P_B/\rho g + (v^2{}_B/2g) + 0.5(v^2{}_B/2g) + h_f$

$0 + 0 + 0 = 1.5 + P_B/\rho g + (v^2{}_B/2g) + 0.5(v^2{}_B/2g) + (fL/d)*(v^2{}_B/2g)$

$0 + 0 + 0 = 1.5 + P_B/\rho g + (v^2{}_B/2g) * \{1 + 0.5 + (fL/d)\}$

$v_B = v_c = 1.26$ m/s

$- 1.5 = P_B/(1000*9.81) + (1.26^2/2*9.81)*\{1 + 0.5 + (0.08*5)/0.1\}$

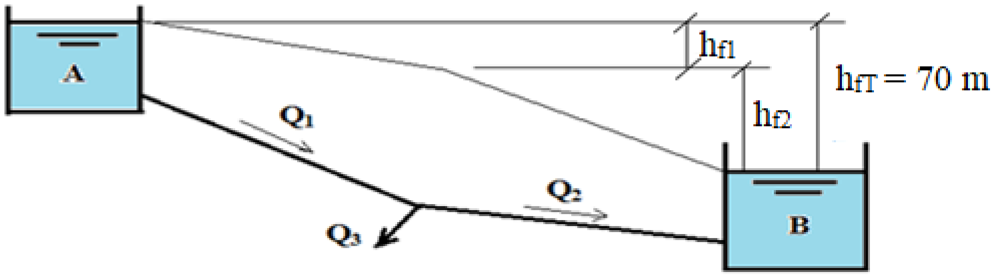
$\therefore P_B = - 28.61$ kN/m^2

Example 6

Two tanks A and B have a 70 m difference in water levels and are connected by a pipe 0.25 m diameter and 6 km long with 0.008 friction coefficient. The pipe is tapped at its mid-point to leak out 0.04 m^3/s flow rate. Minor losses are ignored.

1) Determine the discharge leaving tank A.
2) Find the discharge entering tank B.

Solution



$h_{fT} = h_{f1} + h_{f2}$

$70 = h_{f1} + h_{f2}$

$70 = k_1 Q_1^2 + k_2 Q_2^2$

$$k_1 = k_2 = \frac{8 f L}{\pi^2 g d^5} = \frac{8*0.008*3000}{\pi^2*9.81*0.25^5} = 2{,}032.7$$

$\therefore\ 70 = k_1 Q_1^2 + k_1 Q_2^2$

$Q_1 = Q_2 + Q_3 = Q_2 + 0.04$

$\therefore\ 70 = k_1 (Q_2 + 0.04)^2 + k_1 Q_2^2$

$= k_1 (Q_2^2 + 0.08\ Q_2 + 0.0016) + k_1 Q_2^2$

$= k_1 Q_2^2 + 0.08\ k_1 Q_2 + 0.0016\ k_1 + k_1 Q_2^2$

$= 2\ k_1 Q_2^2 + 0.08\ k_1 Q_2 + 0.0016\ k_1$

$= 4{,}065.4\ Q_2^2 + 162.6\ Q_2 + 3.25$

$0.0172 = Q_2^2 + 0.04\ Q_2 + 0.0008$

$Q_2^2 + 0.04\ Q_2 - 0.0164 = 0$

$$Q2 = \frac{-0.04 \pm \sqrt{(-0.04)^2 - 4(1)(-0.0164)}}{2(1)}$$

$\therefore\ Q_2 = 0.11\ m^3/s$

$Q_1 = Q_2 + 0.04$

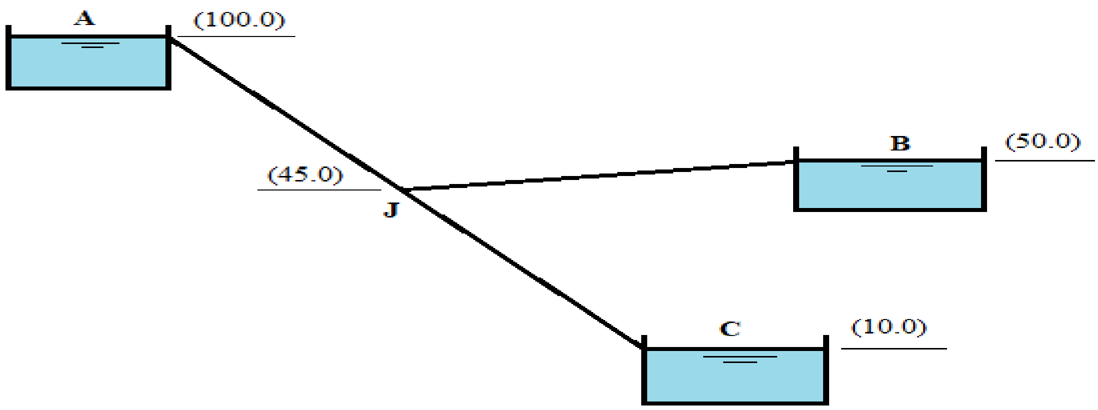
$\therefore\ Q_1 = 0.15\ m^3/s$

Example 7

Three water tanks A, B, and C with water levels (100.00), (50.00), and (10.00) m are connected by pipes AJ, BJ, and CJ to a common joint J with a level of (45.00) m. The three pipes have the same length, diameter, and friction coefficient.

1) Calculate the head at the joint J.
2) Determine the discharge in each pipe.

Solution



Assume, $Q_{AJ} = Q_{JB} + Q_{JC}$

Applying Bernoulli's equation between A and J:

$H_A = H_J + h_{f\,AJ}$

$100 + 0 + 0 = H_J + h_{f\,AJ}$

$100 - H_J = h_{f\,AJ} = K\,Q_{AJ}^2$ *where,* $K = 8\,f\,L / \pi^2\,g\,d^5$

$$Q_{AJ} = \sqrt{\frac{100 - H_J}{K}} = \frac{\sqrt{100 - H_J}}{\sqrt{K}} \quad \text{..............} \quad (1)$$

Similarly, applying Bernoulli's equation between J and B:

$H_J = H_B + h_{f\,JB}$

$H_J - 50 = h_{f\,JB} = K\,Q_{JB}^2$

$$Q_{JB} = \sqrt{\frac{H_J - 50}{K}} = \frac{\sqrt{H_J - 50}}{\sqrt{K}} \quad \text{..............} \quad (2)$$

Also, applying Bernoulli's equation between J and C:

$H_J = H_C + h_{f\,JC}$

$H_J - 10 = h_{f\,JC} = K\,Q_{JC}^2$

$$Q_{JC} = \sqrt{\frac{H_J - 10}{K}} = \frac{\sqrt{H_J - 10}}{\sqrt{K}} \quad \text{..............} \quad (3)$$

Solving equations 1, 2, and 3 by trial and error:

Assumed H_J	**Equation 1**	**Equation 2**	**Equation 3**	**$Q_{AJ} = Q_{JB} + Q_{JC}$**
	$Q_{AJ} * \sqrt{K}$	**$Q_{JB} * \sqrt{K}$**	**$Q_{JC} * \sqrt{K}$**	**$(Q_{JB}+Q_{JC})*\sqrt{K}$**
70	5.48	4.47	7.745	12.216
60	6.325	3.162	7.07	10.233
53	6.855	1.732	6.557	8.289
51	7	1	6.4	7.4
50.5	7.036	0.707	6.364	7.07
50.45	**7.039**	0.671	6.36	**7.031**
50.4	7.043	0.632	6.356	6.988
50	7.071	**0**	6.324	6.324

From the table:

$H_J = 50.45$ m

$Q_{AJ} = 7.039 / (K)^{1/2}$

$Q_{JB} = 0.671 / (K)^{1/2}$

$Q_{JC} = 6.36 / (K)^{1/2}$

It must be noted that if $H_J < 50$, the flow will be from B to J.

Chapter 7

Dimensional Analysis and Similarity

1. Dimensional Homogeneity	*2. Dimensionless Numbers*
3. Bucking-Ham Theorem or π - Theorem	*4. Model Analysis*
5. Hydraulic Similarity	*6. Classification of Models*

7-1 Dimensional Homogeneity

Every term in an equation when reduced to the fundamental dimensions (M, L, T) or (F, L, T) must contain identical powers of each dimension.

For example, $H = Z + (P/\gamma) + (v^2 /2g)$

that is to say: $L = L + (F / L^2) / (F / L^3) + (L^2/T^2) / (L/T^2)$

$\therefore L = L + L + L = L$

7-2 Dimensionless Numbers

Quantities that do not have the fundamental dimensions (M, L, T) or (F, L, T).

For example $2 - \pi - \theta - F_n - R_n$ ----- etc.

7-3 Bucking-Ham Theorem or π - Theorem

For the dimensional homogeneous equation:

$A_1 = f(A_2, A_3, -----, A_n)$

1- $\phi(A_1, A_2, -----, A_n) = 0$ $\therefore$ Number of variables = n

2- Choose 3 repeating variables (m = 3), *as follows:*

The first represents the **geometric properties** (L, w, d, ---).

The second represents **kinematic or fluid properties** (γ, ρ, μ, ---).

The third represents **dynamic or flow properties** (v, a, Q, P, ---).

It must be noted that only one variable is chosen from each group of properties.

Dimensionless numbers must not be chosen.

Let the three repeating variables be A_1, A_2, and A_3.

3- Number of π - terms = n - m, *as follows:*

$\pi_1 = A_1^{a1} * A_2^{b1} * A_3^{c1} * A_4^{-1} = M^0 L^0 T^0$

$\pi_2 = A_1^{a2} * A_2^{b2} * A_3^{c2} * A_5^{-1} = M^0 L^0 T^0$

$\pi_3 = A_1^{a3} * A_2^{b3} * A_3^{c3} * A_6^{-1} = M^0 L^0 T^0$

…

…

…

…

$\pi_{(n-3)} = A_1^{a(n-3)} * A_2^{b(n-3)} * A_3^{c(n-3)} * A_{(n-3)}^{-1} = M^0 L^0 T^0$

4- The values of the powers in each π - term are determined by equating the powers RHS and LHS.

5- $\phi(\pi_1, \pi_2, -----, \pi_{(n-3)}) = 0$

OR $\pi_1 = \psi(\pi_2, \pi_3, -----, \pi_{(n-3)})$

It must be noted that any π - term can be multiplied by or divided by any other π - term or any number or any quantity.

Example 1

The discharge Q through an orifice depends on the pressure P, the density of the fluid ρ, and the diameter of the orifice d.

Determine a general formula for the discharge.

Solution

$\phi(Q, P, \rho, d) = 0$ $\therefore n = 4$

Choose ρ (fluid), Q (flow) and d (geometry) as repeating variables. $\therefore m = 3$

Number of π - terms = m - n = 4 - 3 = 1

$\pi_1 = \rho^a Q^b d^c P^{-1} = M^0 L^0 T^0$

$(M L^{-3})^a (L^3 T^{-1})^b (L)^c (M L^{-1} T^{-2})^{-1} = M^0 L^0 T^0$

$M^{a-1} = M^0$ a - 1 = 0 **a = 1**

$L^{-3a+3b+c+1} = L^0$ -3a + 3b + c + 1 = 0 .... (1)

$T^{-b+2} = T^0$ - b + 2 = 0 **b = 2**

In equation (1), -3 + 6 + c +1 = 0 **c = - 4**

$\therefore$ $\pi_1 = \rho Q^2 d^{-4} P^{-1} = (\rho / P) * (Q / d^2)^2$

$\phi [(\rho / P) * (Q / d^2)^2] = 0$

$\psi [(\rho / P)^{1/2} * (Q / d^2)] = 0$

$(Q / d^2) = K / (\rho / P)^{1/2}$ *where* K is a constant.

$Q = K d^2 / (\rho / P)^{1/2} = K d^2 (P / \rho)^{1/2}$

But $P = \rho g h$

Then, $Q = K d^2 (g h)^{1/2}$

<u>*Multiplying both sides by (4 * π * 2^1/2),*</u>

$$Q = K\, d^2 \sqrt{g\, h} * \frac{4 * \pi * 2^{\frac{1}{2}}}{4 * \pi * 2^{\frac{1}{2}}}$$

$$Q = \frac{4\, K}{2^{\frac{1}{2}}\, \pi} \sqrt{2g\, h} * \frac{\pi * d^2}{4}$$

$\therefore Q = C_d A (2 g h)^{1/2}$ *where,* $Cd = (2^{3/2} K / \pi)$

7-4 **Model Analysis**

It is a scientific method to predict the performance of hydraulic structures, systems, and machines. A model is prepared and tested in a laboratory for the working and behavior of the proposed hydraulic system. The hydraulic system, for which a model is prepared, is known as the prototype.

7-5 **Hydraulic Similarity**

For the model analysis, there should be a complete similarity between the prototype and its model. This similarity is known as hydraulic similarity or hydraulic similitude.

Hydraulic similarity includes three types:

A- Geometric similarity

The prototype and its model are identical in shape but are different in size. The ratios of all corresponding linear dimensions are equal.

Scale (linear) ratio $L_r = L_m / L_p = d_m / d_p = y_m / y_p$

Where, L: length, d: diameter, and y: depth.

B- Kinematic similarity

The prototype and its model have identical motions or velocities. The ratios of the corresponding velocities at corresponding points are equal.

Velocity ratio $v_r = v_{1m} / v_{1p} = v_{2m} / v_{2p} =$ ……………

C- Dynamic similarity

The prototype and its model have identical forces. The ratios of the corresponding forces acting at corresponding points are equal.

Force ratio $F_r = F_{1m} / F_{1p} = F_{2m} / F_{2p} =$ ……………

Forces can be divided into external forces and internal forces.

External forces include

1- Pressure force (F = P * A).

2- Gravity force (F = m * g).

Internal forces include:

1- Inertia force (F = m * a)

2- Viscosity force (F = τ * A= (μ v / L) (L^2) = μ v L)

3- Surface tension force (F = σ * L).

4- Elasticity force (F = K * A).

In general, the force ratio is constant and equal for the different types of forces for a prototype and its model. Some forces may not act or may be very small (neglected).

The ratio of the inertia force to any other present (predominant) force provides the known dimensionless numbers, which will be used to solve problems of model analysis.

However, only two forces are going to be discussed.

1- Gravity force is present

Examples are flow through open channels, flow over weirs, and surface waves.

$$\frac{F_{im}}{F_{ip}} = \frac{F_{gm}}{F_{gp}}$$

$$\frac{F_{im}}{F_{gm}} = \frac{F_{ip}}{F_{gp}}$$

$$\frac{\rho_m\, L_m^2\, v_m^2}{\rho_m\, L_m^3\, g_m} = \frac{\rho_p\, L_p^2\, v_p^2}{\rho_p\, L_p^3\, g_p}$$

$$\frac{v_m^2}{L_m\ g_m} = \frac{v_p^2}{L_p g_p}$$

$$\frac{v_m}{\sqrt{L_m\ g_m}} = \frac{v_p}{\sqrt{L_p\ g_p}}$$

That is to say that $\mathbf{F_m = F_p}$, where F is the Froud number.

$$\frac{v_m\ \sqrt{L_p\ g_p}}{v_p\ \sqrt{L_m\ g_m}} = 1$$

$$\frac{v_m}{v_p\ \sqrt{\frac{L_m}{L_p}\ \frac{g_m}{g_p}}} = 1$$

$$\frac{v_r}{\sqrt{L_r\ g_r}} = 1$$

<u>Which is Froud's law.</u>

If **gr = 1,** **then** $\mathbf{v_r = \sqrt{L_r}}$

2- ***<u>Viscous force is present</u>***

Examples are the flow in pipes, hydraulic measuring devices (flow meters), and hydraulic machines (pumps and turbines).

$$\frac{F_{im}}{F_{ip}} = \frac{F_{vm}}{F_{vp}}$$

$$\frac{F_{im}}{F_{vm}} = \frac{F_{ip}}{F_{vp}}$$

$$\frac{\rho_m\ L_m^2\ v_m^2}{\mu_m\ L_m\ v_m} = \frac{\rho_p\ L_p\ v_p}{\mu_p}$$

$$\frac{\rho_m\ L_m\ v_m}{\rho_m\ \nu_m} = \frac{L_p\ v_p}{\nu_p}$$

$$\frac{L_m\ v_m}{\nu_m} = \frac{L_p\ v_p}{\nu_p}$$

That is to say that $\mathbf{R_m = R_p}$, where R is Reynold's number.

$$\frac{L_m \, v_m \, \nu_p}{L_p \, v_p \, \nu_m} = 1$$

$$\frac{L_r \, v_r}{\nu_r} = 1$$

Which is Reynolds's law.

Then, $\mathbf{v_r = \upsilon_r / L_r}$

7-6 Classification of Models

A- Undistorted Models

The geometric similarity is the same for both horizontal and vertical linear dimensions.

B- Distorted Models

The geometric similarity is different for both horizontal and vertical linear dimensions. Scale ratios will be L_{rh} and L_{rv}. For example, studying a river basin.

Example

A model for a spillway is built in a laboratory where the maximum capacity of the pump is 9 cfs. The prototype has 300 cfs maximum discharge and a 5 ft head on the crest.

1) Determine the scale ratio for the model.
2) Calculate the head on the crest of the model.
3) Find the time in the model corresponding to 36 hours in the prototype.
4) Determine the loss of power in the prototype corresponding to the observed 0.05 HP in the model.

Solution

1) $Q_r = Q_m / Q_p = 9 / 300 = 3 / 100$

$Q_r = A_m v_m / A_p v_p = L_m^2 v_m / L_p^2 v_p = L_r^2 v_r$

The case is a spillway, i.e. flow through open channels, so the gravity force is the present force. Thus, Froud's law is applied.

$$\frac{v_r}{\sqrt{L_r\, g_r}} = 1$$

$g_r = 1$, then $v_r = (L_r)^{1/2}$

$Q_r = v_r * A_r = (L_r)^{1/2} * (Lr)^2 = (L_r)^{5/2}$

$L_r = (Q_r)^{2/5} = (3 / 100)^{2/5} = 0.25 = 1/4$

2) $L_r = h_m / hp$

$h_m = h_p * L_r = 5 * 0.25 = 1.25$ ft

3) $T_r = L_r / v_r = (L_r) / (L_r)^{1/2} = (L_r)^{1/2} = (0.25)^{1/2} = T_m / T_p$

$T_m = T_p * T_r = 36 * (0.25)^{1/2} = 18$ hours

4) $P_r = P_m / P_p = (\gamma_m Q_m h_m / 75) / (\gamma_p Q_p h_p / 75)$

$= (\gamma_r Q_r h_r) = (1) (L_r)^{5/2} (L_r) = (L_r)^{7/2}$

$P_p = P_m / P_r = 0.05 / (0.25)^{7/2} = 6.4$ HP

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